PG Odd Semester (CBCS) Exam., December—2017

ECONOMICS

(1st Semester)

Course No. : ECOCC–103

(Mathematical Methods for Economic Analysis)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

Unit—1

1. *(a)* Let $A \quad \{5, 8, 9\}$ and $B \quad \{10, 16, 15\}$. Define relation R from A to B by—

(i) $R \quad \{(x, y): 2x \quad y \quad 0,$ where $x \quad A,$ $y \quad B\};$

(ii) $R \quad \{(x, y): x \quad y$ odd number, where $x \quad A, y \quad B\};$

(iii) $R \quad \{(x, y): x \quad y \quad 80,$ where $x \quad A,$ $y \quad B\}.$

Which of these relations is also mathematical functions? Why?

(b) "A polynomial function is a special case of rational function." Comment.

(c) Given the supply and demand functions for two goods, A and B.

Good A : $\{Q_{da} \quad 30 \quad 8P_a \quad 2P_b$
$\{Q_{sa} \quad 15 \quad 7P_a$

Good B : $\{Q_{db} \quad 28 \quad 4P_a \quad 6P_b$
$\{Q_{sb} \quad 12 \quad 2P_b$

(i) Write down the equilibrium condition for each good. Hence, deduce two equations in P_a and P_b.

(ii) Use Cramer's rule to find the equilibrium prices and quantities for goods A and B.

(3+2)+3+(2+4)=14

2. *(a)* Given

$$z \quad \frac{(3x \quad 11y)^3}{2x \quad 6y}$$

Find all the first-order partial derivatives.

(b) Given $z \quad x^2 \quad 8xy \quad y^3$, where $x \quad 3t$, $y \quad 1 \quad t$. Draw the channel map for this function. Also find dz/dt.

(c) What is the basic difference between a differential equation and a difference equation?

(d) Given

$$\frac{dy}{dt} \quad 5y \quad 0$$

Find the inter-temporal equilibrium level of $y(t)$. Is the system dynamically stable? Comment. 4+4+2+4=14

UNIT—2

3. (a) Why do we need first-order derivative of a function to be zero for optimization? Why is it not sufficient?

(b) Find the point elasticity of supply E_s from the supply function $Q \quad P^2 \quad 7P$. Also examine whether the supply is elastic at $P \quad 2$.

(c) If the investment flow is $I(t) \quad 9000\sqrt{t}$, calculate (i) the capital formation from the end of the first year to the end of the fourth year and (ii) the number of years required before the capital stock exceeds ₹ 1,00,000.

(2+2)+(3+1)+(3+3)=14

4. (a) If the demand function for a profit maximizing monopolist is $P \quad 274 \quad Q^2$ and MC $\quad 4 \quad 3Q$, find the consumers' surplus.

(b) Do you think consumption functions with lag income as explanatory variable is more relevant than traditional consumption function? Give argument in support of your answer.

(c) Assume a simple national income model

$$Y_t \quad C_t \quad I_t$$

where $C_t \quad 5000 \quad 0\ 8Y_{t\ \ 1}$ and $I_t \quad 5000$.

(i) Write the national income equation as a difference equation in Y.

(ii) Solve the difference equation.

(iii) If $Y_0 \quad 100000$, find the particular solution. Plot the time path for $t \quad 0$ to $t \quad 5$. 5+2+(1+3+3)=14

UNIT—3

5. (a) Show that the function $y \quad x \quad 1/x$ (with $x \quad 0$) has two relative extrema, one a maximum and the other a minimum. Is the 'minimum' larger or smaller than the 'maximum'? How is this paradoxical result possible?

(b) Mr. Sen wishes to mark out a rectangular flowerbed, using a wall of his house as one side of the rectangular. The other three sides are to be marked by wire netting, of which he has only 64 ft available. What are the length L and width W of the rectangle that would give him the largest possible planting area? How do you make sure that your answer gives the largest, not the smallest area?

(c) Show that the law of diminishing marginal productivity holds for the production function $Q \quad 12\ K^{0\ 4}L^{0\ 4}$. What is the nature of returns to scale of this production function?

(d) A multiplant monopoly operates two plants whose total cost schedules are

$$\text{TC}_1 \quad 8\ 5 \quad 0\ 03q_1^2, \quad \text{TC}_2 \quad 5\ 2 \quad 0\ 04q_2^2$$

If it faces the demand schedule

$$P \quad 60 \quad 0\ 04q$$

where $q \quad q_1 \quad q_2$, derive the expression of profit function. (3+2)+(2+2)+2+3=14

6. *(a)* Graphically illustrate the difference between constraint and unconstraint of limitations.

(b) Given the demand function

$$Q \quad 500 \quad 3P \quad 2P_A \quad 0\ 01Y$$

where $P \quad 20$, $P_A \quad 30$ and $Y \quad 5000$.

(i) Find the income elasticity of demand.

(ii) If income rises by 5%, calculate the corresponding percentage change in demand. Is the good inferior or normal?

(c) A monopolist produces a single good X but sells in two separate markets. The demand function for each market is

$$P_1 \quad 50 \quad 4x_1 \text{ and } P_2 \quad 80 \quad 3x_2$$

The cost function is $\text{TC} \quad 120 \quad 8x$, where $x \quad x_1 \quad x_2$. Should the firm charge discriminatory prices in these two markets? 3+(2+2)+7=14

UNIT—4

7. *(a)* Explain the concept of Lagrange multiplier with the help of an example.

(b) Write the Lagrangian function and the first-order condition for stationary values (without solving the equations) for the following function :

$$\text{Optimize } Z \quad x^2 \quad 2xy \quad yw^2$$

subject to

$$x \quad w \quad 8$$

$$2x \quad y \quad w^2 \quad 24$$

(c) The prices of inputs K and L are given as ₹120 per unit and ₹30 per unit respectively, and a firm operates with the production function, $Q \quad 25K^{0\ 5}L^{0\ 5}$. Calculate the minimum cost of producing 1250 units of output. Also check sufficient conditions of optimization. 4+2+8=14

8. (a) What is a feasible solution? Sketch the feasible region for the following inequalities :

$$25x \quad 80y \quad 800$$
$$20x \quad 5y \quad 220$$
$$x \quad y \quad 10$$

Also state whether each constraint is acting as a limitation.

(b) A small manufacturer produces two kinds of good, A and B, for which demand exceeds capacity. The production costs for A and B are ₹6 and ₹3, respectively, each, and the corresponding selling prices are ₹7 and ₹4. In addition, the transport costs are 20 paisa and 30 paisa for each good of type A and B, respectively. The conditions of a bank loan limit the manufacturer to maximum weekly production costs of ₹2,700 and maximum weekly transport costs of ₹120. Assuming that the manufacturer has to arrange production to maximize profit, formulate the linear programming problem (LPP).

(c) Solve the following problem by simplex method :

Maximize $Z \quad 55x \quad 25y$

subject to

$$6x \quad 2y \quad 840$$
$$2x \quad y \quad 300$$
$$x \quad y \quad 250$$
$$x, y \quad 0$$

4+3+7=14

UNIT—5

9. (a) Show that existence of the dominant strategy equilibrium implies existence of a Nash equilibrium but the reverse is not true.

(b) Two computer firms, A and B, are planning to market network systems for office information management. Each firm can develop either a fast, high-quality system (H) or a slower, low-quality system (L). Market research indicates that the resulting profits to each firm for the alternative strategies are given by the following payoff matrix :

		Firm B: H	Firm B: L
Firm A	H	30, 30	50, 35
	L	40, 60	20, 20

(i) If both firms make their decisions at the same time and follow maximin (low-risk) strategies, what will the outcome be?

(ii) Suppose both firms try to maximize profits, but firm *A* has a head start in planning, and can commit first. Now what will the outcome be? What will the outcome be if firm *B* has the head start in planning and can commit first? 5+(3+6)=14

10. (a) What is meant by 'first-mover advantage'? Give an example of a gaming situation with a first-mover advantage.

(b) What is the meaning of 'tit for tat' in game theory? What conditions are usually required for tit for tat strategy to be the best strategy?

(c) Given

		Firm B	
		Low Price	*High Price*
Firm A	*Low Price*	1, 1	3, 1
	High Price	1, 3	2, 2

What would be the tit for tat strategy for the first five of an infinite number of games, if firm *A* starts by cooperating but firm *B* does not cooperate in the next period? (2+4)(2+2)+4=14

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