PG Odd Semester (CBCS) Exam., December—2017

ECONOMICS

(3rd Semester)

Course No. : ECOCC–305

(Mathematical Economics—I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Assess the role of the Lagrangian multiplier in the case of a consumer maximizing her utility subject to a constraint.

(b) Establish the relationship between the Lagrangian multiplier of a utility maximization problem and its dual problem.

(c) Given that

$$R \quad \frac{(y \quad b)}{(x \quad a)}$$

is the MRS_{yx} for a consumer throughout the indifference map. Find the exact form of the utility function. 5+7+2=14

2. *(a)* Consider a consumer who has a fixed income and consumes two goods at fixed prices. Determine the compensated demand function for the two goods.

(b) A consumer has been observed to purchase 20 units of goods x_1 and 10 units of goods x_2 at prices ₹ 2 per unit and ₹ 6 per unit respectively. She has been also observed to purchase 18 units of x_1 and 4 units of x_2 at prices ₹ 3 and ₹ 5 per unit respectively. Whether the behaviour of the consumer is consistent with the weak axiom of revealed preference? Verify.

(c) State and prove Roy's identity for a two-commodity model. 5+4+5=14

UNIT—II

3. *(a)* Suppose $f(x_1, x_2, \cdots, x_n)$ is a homogeneous function of degree r. Then show that

$$x_1 \frac{f}{x_1} \quad x_2 \frac{f}{x_2} \quad \cdots \quad x_n \frac{f}{x_n}$$

$$rf(x_1, x_2, \cdots, x_n)$$

(b) Prove that for a Cobb-Douglas production function of the form $Q \quad AL \quad K$, where and are partial elasticities of output with respect to inputs, as well as distributive parameters.

(c) A firm has a production function of the form $Q \quad \sqrt{4A} \quad \sqrt{9B}$. A and B are variable inputs. Input prices are equal. Show that in equilibrium, the firm will use the same amount of both inputs.

4+(3+3)+4=14

4. *(a)* Consider the production function $Q \quad Ax_1 \, x_2^1$. If input prices are r_1 and r_2, show that the expansion path is given by

$$(1 \quad) r_1 x_1 \quad r_1 r_2 \quad 0$$

(b) Show how you can derive a firm's total cost function if you know its production function and the price of the inputs.

(c) The partial elasticities of two inputs x and y are given by and respectively. Determine the production function.

3+8+3=14

UNIT—III

5. *(a)* If the monopolist has the total cost function $C \quad ax^2 \quad bx \quad c$ and if the demand function is $p \quad x^2$, show that output for maximum net profit is

$$x \quad \frac{\sqrt{\{a^2 \quad 3 \quad (\quad b)\}} \quad a}{3a}$$

(b) A sitar manufacturer can sell x sitars per week at p rupees each, where $5x \quad 375 \quad 3p$. The cost of production is $\{500 \quad 13x \quad 1/5x^2\}$ rupees. Find how many sitars should manufacture for maximum profit and what is that profit.

(c) Find the profit maximizing output level, given $x \quad 200 \quad 10p$, $AC \quad 10 \quad x/25$.

6+5+3=14

6. *(a)* A firm produces x units of a product per week at a total cost of

$$₹(1/3x^3 \quad 15/2x^2 \quad 10x \quad 100)$$

(i) Find the output levels at which the marginal cost, average variable cost and average total cost attain their respective minima.

(ii) If the market price is fixed at ₹ 110 per unit, find the equilibrium output of the firm. Does it cover the total cost? Give reasons for your answer.

(b) Find the elasticities of demand and supply at equilibrium price for demand function $p \sqrt{100 x^2}$ and supply function $x 2p 10$, where p is price and x is quantity. (4+5)+5=14

UNIT—IV

7. (a) Check Walrasian and Marshallian static stability conditions for the following demand-supply system (symbols have their usual meanings) :

Demand : $p 10 0 8q$

Supply : $p 5 0 05q$

(b) Show how an extra equation in the Cassel-Walras general equilibrium model can be eliminated to make it an exactly determinate system.

(c) Suppose that a government is a monopolist in some commodity and has full information about market demand given by

$$Q \frac{m}{n} \frac{1}{n} p$$

with (m, n) 0. Its cost function is given by $C Q$, with 0 and m . Compute the gain in consumer surplus when the government moves from monopoly to competitive pricing. 4+3+7=14

8. (a) Find profit-maximising outputs and maximum profit for the competitive firm producing two goods, where p_1 12 and p_2 18 are the given prices and $C 2Q_1^2 2Q_2^2 Q$ is the cost function.

(b) In a standard duopoly model for a non-differentiated good, the following information is given to you :

Market demand function :

$$p (q_1 q_2)$$

Cost function : $C_i cq_i i$ 1, 2 and 0 .

Here q_1 and q_2 are supply quantities of the same good by firm–1 and firm–2 respectively. Derive the reaction functions for each firm and find the 'Cournot-Nash equilibrium' outputs and profit. How does this model behave when there are large number of identical firms? 4+10=14

UNIT—V

9. Derive the Lucas supply function under imperfect information for producers expressing deviation from normal output as an increasing function of surprise inflation. Can you establish monetary neutrality under this setup? If so, how? 10+4=14

10. Show how mistaken price expectations of workers under optimizing behaviour of households and firms can generate the Friedman aggregate supply rule. Hence explain how Friedman's result can justify the natural rate of hypothesis. 10+4=14

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