PG Odd Semester (CBCS) Exam., December—2018

ECONOMICS

(1st Semester)

Course No. : ECOCC–103

(Mathematical Methods for Economics)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Let $A \quad \{1, 2, 3, 4, 5, 6\}$. Define a relation R from A to A by

$$R \quad \{(x, y) : y \quad x \quad 1\}$$

Write down the domain, codomain and rank of R. 2+3=5

(b) Examine each of the following relations and state in each case, giving reasons, whether it is a function or not : 3

(i) $R \quad \{(2, 1), (3, 1), (4, 2)\}$

(ii) $R \quad \{(2, 2), (2, 4), (4, 4)\}$

(iii) $R \quad \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)\}$

(c) Given the IS equation

$$0 \quad 3Y \quad 100i \quad 252 \quad 0$$

and the LM equation

$$0 \quad 25Y \quad 200i \quad 176 \quad 0$$

Find the equilibrium level of income (Y) and rate of interest (i). 6

2. *(a)* Given $y \quad (16x \quad 3)^{2}$, use chain rule to find $\frac{dy}{dx}$. Then rewrite the function as $y \quad \frac{1}{(16x \quad 3)^{2}}$ and find $\frac{dy}{dx}$ by the quotient rule. Are the answers identical? 5

(b) Evaluate : 2×2=4

(i) $(x \quad 3)(x \quad 1)^{\frac{1}{2}} dx$

(ii) $x \ln x \, dx (x \quad 0)$

(c) Use the formula for a general solution to solve the following equation :

$$\frac{dy}{dt} \quad 6y \quad 18$$

Is the system dynamically stable? 5

UNIT—II

3. *(a)* Draw a curve which is not strictly concave. 2

(b) Which of the following quadratic functions are strictly convex? 2×3=6

(i) $y \quad 9x^2 \quad 4x \quad 8$

(ii) $u \quad 9 \quad 2x^2$

(iii) $w \quad 3x^2 \quad 39$

(c) For the following functions—

(i) find the critical values;

(ii) test for concavity to determine relative maxima or minima;

(iii) check for inflection points;

(iv) evaluate the function at the critical values and inflection points : 6

$$y \quad x^3 \quad 18x^2 \quad 96x \quad 80$$

4. *(a)* Given the demand function

$$P_d \quad 113 \quad Q^2$$

and the supply function

$$P_s \quad (Q \quad 1)^2$$

Find the producer's surplus. 6

(b) Given the equilibrium condition that

$$I_t \quad S_t$$

where $I_t \quad 2 \ 5(Y_t \quad Y_{t \ 1})$ and $S_t \quad 0 \ 1Y_t$.

(i) Write the equilibrium equation as a difference equation in Y_t.

(ii) Solve the difference equation, given $Y_0 \quad 8$.

(iii) Plot the time path for $t \quad 0$ to $t \quad 7$. Is the system dynamically stable?

2+3+3=8

UNIT—III

5. *(a)* Compare and contrast between local and global optimizations. 4

(b) Find the first-order partial derivatives of the following functions : 3×2=6

(i) $q \quad 20x^{0 \ 6}y^{0 \ 2}z^{0 \ 3}$

(ii) $z \quad (x^3 \quad 7y^2)^4$

(c) Find the extreme values of the following function :

$$Z \quad x^2 \quad xy \quad 2y^2 \quad 3$$

Also examine if the function attains maxima or minima at the extreme value. 4

6. *(a)* A firm produces two products which are sold in two separate markets with the demand schedule

$$p_1 \quad 600 \quad 0 \ 3q_1, \ p_2 \quad 500 \quad 0 \ 2q_2$$

Production costs are related and total cost is

$$TC \quad 16 \quad 1 \ 2q_1 \quad 1 \ 5q_2 \quad 0 \ 2q_1q_2$$

If the firm wishes to maximize total profit, how much of each product should it sell? What will be the maximum profit? 4+2=6

(b) Given

$$Q_1 \quad 50 \quad 4P_1 \quad 3P_2 \quad 2P_3 \quad 0\ 001\,Y$$

where $P_1 \quad 5$, $P_2 \quad 7$, $P_3 \quad 7$, $Y \quad 11000$ and $Q_1 \quad 26$. Estimate the effect on Q_1 of a 10% increase for each of the other goods individually. 4

(c) Let an isoquant for the output level $Q \quad 2144$ is

$$16K^{1/4}L^{3/4} \quad 2144$$

Find the slope of the isoquant $\frac{dK}{dL}$. 4

UNIT—IV

7. *(a)* If the Lagrangian function is written as

$$Z \quad f(x, y) \quad [g(x, y) \quad C]$$

rather than

$$Z \quad f(x, y) \quad [C \quad g(x, y)]$$

does it make any difference to the equilibrium value of the variables? Give the new interpretation, if any. 4

(b) Write the Lagrangian function and the first-order condition for stationary values (without solving the equations) for the following function : 3

$$Z \quad x_1^2 \quad 2x_1x_2 \quad x_2x_3^3$$

subject to

$$2x_1 \quad x_2 \quad x_3^2 \quad 24$$
$$x_1 \quad x_3 \quad 8$$

(c) A firm has the production function $Q \quad K^{0\,5}L^{0\,5}$ and buys input K at ₹ 12/unit and input L at ₹ 3/unit and has a budget of ₹ 600. Find the input combination that will maximize output. Also check the second-order condition for optimization. 7

8. *(a)* What is the advantage of linear programming method over Lagrange multiplier method in optimization problem? Briefly discuss. 3

(b) A company produces two types of iron gates. The number of man hours required to produce each type of gate

along with the maximum number of hours available are given below :

	Welding	*Finishing*	*Admin*	*Selling price*
Type 1 gate	6	2	1	120
Type 2 gate	2	1	1	95
Maximum hours available	840	300	250	

(i) Express the information given on input requirement in terms of inequality constraints.

(ii) Graph the inequality constraints and shade in the feasible region.

2+5=7

(c) Given the following primal, formulate its dual : 4

Maximize, $Z \quad 4x \quad 9y$

subject to

$$5x \quad 3y \quad 30$$
$$7x \quad 2y \quad 28$$
$$x, y \quad 0$$

UNIT—V

9. *(a)* Explain the meaning of a Nash equilibrium. How does it differ from an equilibrium in dominant strategies? 5

(b) Determine the optimal strategies for the two players *A* and *B* and find the value of the game from the following pay-off matrix : 6

Player—A

$$\textit{Player—B} \begin{bmatrix} 3 & -1 & 4 & 2 \\ -1 & -3 & -7 & 0 \\ 4 & -6 & 2 & -9 \end{bmatrix}$$

(c) If player-1 has 30 strategies and player-2 has 25, how many possible outcomes of the game will these be? Briefly explain. 3

10. *(a)* Illustrate the concept of 'coordination' game with special reference to battle of soxes. 8

(b) A contractor says that he intends to 'low-ball the bid and make up for it on change orders'. What does he mean? Discuss. 6

★ ★ ★