PG Odd Semester (CBCS) Exam., December—2018

ECONOMICS

(1st Semester)

Course No. : ECOCC–103

(Mathematical Methods for Economics)

$$\dfrac{\textit{Full Marks} : 70}{\textit{Pass Marks} : 28}$$

Time : 3 hours

The figures in the margin indicate full marks
for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Examine each of the following relations and state in each case, giving reasons whether it is a function or not :
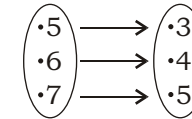
   (i) $R = \{(2, 1), (3, 1), (4, 2)\}$

   (ii) $R = \{(2, 2), (2, 4), (3, 3), (4, 4)\}$

   (iii) $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)\}$

   Also find the range and domain of these functions, if any.   2+2+2=6

(b) The following figure shows a relationship between the sets P and Q :

Write this relation *(i)* in set-builder form and *(ii)* in roster form.   2+2=4

(c) "All functions are also relations but the reverse is not true." Justify the statement with a suitable illustration.   4

2. *(a)* Find the first-order partial derivatives of the following functions :   3+3=6

   (i) $q = 20x^{0·6}y^{0·2}z^{0·3}$

   (ii) $z = (x^3 - 7y^2)^4$

(b) Draw the channel map and find the total derivative dz/dx for the following function :   3

   $z = (13x - 18y)^2$, where $y = x - 6$

(c) Given $Q_1 = 50 - 4P_1 - 3P_2 - 2P_3 + 0·001Y$, where $P_1 = 5$, $P_2 = 7$, $P_3 = 3$ and $Y = 11000$, $Q = 26$.

   (i) Identify the relationship between good 1 and the other two goods.

   (ii) Calculate the effect on Q_1 of a 10% increase for each of the other goods individually.   2+3=5

(3)

UNIT—II

3. *(a)* Evaluate :   3+3=6

    (i) $\int \ln x \, dx$

    (ii) $\int_{1}^{2} (2x^3 - 1)^2 (6x^2) \, dx$

(b) The definite integral $\int_{0}^{b} f(x) \, dx$ is said to represent an area under a curve. Does this curve refer to the graph of the integrand $f(x)$, or of the primitive function $F(x)$? If we plot the graph of the $F(x)$ function, how can we show the given definite integral on it by an area, a line segment, or a point?   3+1=4

(c) Given the MPC $C'(Y) = 0 \cdot 8 - 0 \cdot 1 Y^{-1/2}$ and the information that $C = Y$, when $Y = 100$, find the consumption function $C(Y)$.   4

4. *(a)* Use the formula for a general solution to solve the following equation :

$$dy \, / \, dt + 6y = 18$$

Is the system dynamically stable? Comment.   4

(b) Graphically discuss the concept of explosive oscillation in the context of cobweb model.   4

(4)

(c) Given the equilibrium condition that $I_t = S_t$, where $I_t = 2 - 5(Y_t - Y_{t-1})$ and $S_t = -0 \cdot 1 Y_t$

    (i) Write the equilibrium equation as a difference equation in Y_t.

    (ii) Solve the difference equation, given $Y_0 = 8$.

    (iii) Plot the time path for $t = 0$ to $t = 7$.   1+2+3=6

UNIT—III

5. *(a)* Draw the graph of a function $f(x)$ such that $f'(x) = 0$ and the graph of a function $g(x)$ such that $g'(x) = 0$. Summarize in one sentence the essential difference between $f(x)$ and $g(x)$ in terms of the concept of stationary point.   4

(b) Draw *(i)* a concave curve which is not strictly concave and *(ii)* a curve which qualifies simultaneously as a concave curve and a convex curve.   2+2=4

(c) Given the following function :

$$Y = x^3 - 18x^2 + 96x - 80$$

    (i) Find the critical values.

    (ii) Test for concavity to determine relative maxima or minima.

    (iii) Check for inflection points.   2+2+2=6

(5)

6. *(a)* A purely competitive firm has a single variable input L (labour), with the wage rate W_0 per period. Its fixed inputs cost the firm a total of F dollars per period. The price of the product is P_0.

 (i) Write the production function, revenue function, cost function and profit function of the firm.

 (ii) What is the first-order condition for profit maximization? Give this condition an economic interpretation.

 (iii) What economic circumstances would ensure that profit is maximized rather than minimized? 4+2+2=8

(b) What combination of good x and good y should a firm produce to minimize costs when the joint cost function is $C = 6x^2 + 10y^2 + xy + 30$ and the firm has a production quota of $x + y = 34$? Also estimate the effect on costs if the production quota is reduced by 1 unit.

5+1=6

UNIT—IV

7. *(a)* What is the advantage of linear programming method over Lagrange's multiplier method in optimization problem? 3

(6)

(b) A company produces two types of wrought iron gates. The number of man-hours required to produce each type of gate along with the maximum number of hours available is given in the following table : 2+5=7

Requirements for Gates I and II

	Welding	Finishing	Admin	Selling price
Type–1 gate	6	2	1	120
Type–2 gate	2	1	1	95
Max hours available	840	300	250	

 (i) Express the information on input requirements in terms of inequality constraints.

 (ii) Graph the inequality constraints and shade in the feasible region.

(c) What will happen in the graph of a linear program with two choice variables, if the following are true?

 (i) The coefficient of x_1 in the objective function is zero.

 (ii) The first two constraints are respectively

$$ax_1 + bx_2 \le c$$
$$ax_1 + bx_2 \ge c$$

2+2=4

(7)

8. *(a)* Given the following primal; formulate its dual : 3

Minimize

$$C = x_1 + 7x_2$$
$$x_1 + x_2 \geq 5$$
$$x_2 \geq 4$$
$$2x_1 + 3x_2 \geq 9$$
$$x_1, x_2 \geq 0$$

(b) Rewrite the dual of the last problem in vector form and give geometrical interpretation of the non-negativity constraints. 2+2=4

(c) Solve the following problem by the simplex method : 7

Maximize $4x + 9y$

subject to

$$5x + 3y \leq 30$$
$$7x + 2y \leq 28$$
$$x, y \geq 0$$

UNIT—V

9. *(a)* What is a dominant strategy? Illustrate. Why is an equilibrium stable in dominant strategies? 3+2=5

(8)

(b) If player 1 has 30 strategies and 2 has 25, how many possible outcomes of the game will there be? Briefly explain. 2

(c) Two firms are in the chocolate market. Each can choose to go for the high end of the market (high quality) or the low end (low quality). Resulting profits are given by the following pay-off matrix :

		Firm–2	
		Low	High
Firm–1	Low	–20, –30	200, 600
	High	100, 800	50, 50

(i) What outcomes, if any, are Nash equilibria?

(ii) If the manager of each firm is conservative and each follows a maximin strategy, what will be the outcome?

(iii) What is the cooperative outcome?

(iv) Which firm benefits most from the cooperative outcome? How much would that firm need to offer the other to persuade it to collude? 2+2+1+2=7

10. *(a)* Given the following pay-off matrix :

$$\text{Firm–2}$$

		S_1	S_2
	S_1	(2, 2)	(3, 6)
Firm–1			
	S_2	(6, 3)	(2, 2)

(i) Represent the above game through its extensive form for two different situations—Firm–1 as the first mover in first situation, and Firm–2 as the first mover in second situation.

(ii) Do you think it is the first mover that gains from the game? Justify.

$$4+2=6$$

(b) Illustrate the concept of coexistence game with special reference to inter-generation conflict over saving.   8

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