Chapter Three

MODELS, METHODOLOGY AND DATA

This chapter deals with the methodology adopted for the present study. The analytical foundations of measurement of economic efficiency, formulation of the stochastic production frontier and collection of primary data are articulated in detail. The analytical microeconomic foundations of measurement of technical efficiency are explained in the first sub-section, that is, section 3.1. Concepts of production frontier, input and output based measures of technical efficiency are elaborated in this section using standard microeconomic tools. Some elementary real analysis is used following graduate level standard microeconomic texts. The second subsection, that is, section 3.2 is dedicated to the concept of the stochastic production frontier and its conceptual development since its inception in 1977. The next four sections, that is, sections 3.3, 3.4, 3.5 and 3.6 are devoted respectively to the specific econometric strategy adopted in the study. The subsection 3.7 deals with the estimation of Cobb-Douglas Cost Frontier used to estimate firm level cost efficiency. Section 3.8 deals with the methodological issues relating to the measurement of Total Factor Productivity Growth (TFPG) which is further subdivided into two subsections, that is, 3.8.1 and 3.8.2. The first subsection discusses the parametric approach of measuring TFPG whereas the second subsection is related to the non-parametric approach. Finally, the nature and sources of data along with collection are narrated in section 3.9.

3.1. Analytical Foundations of Measurement of Economic Efficiency

Prior to describing the econometric strategy adopted to measure firm level economic efficiency or inefficiency it is essential to present the conceptual (microeconomic) framework of economic efficiency. We begin with the physical structure of different production technologies with the help of graphical analysis. A production technology that transferring inputs $x=(x_1,x_2,.......,x_N)\in R_+^N=\{x:x\in R_+^N,x\geq 0\}$ into output $y=(y_1,y_2,....,y_M)\in R_+^N$ can be represented by the output correspondence L or the graph of the technology GR. The output correspondence P: $R_+^N\rightarrow 2^{R_+^N}$ $[2^{R_+^N}\Rightarrow\{A:A\subseteq R^M\}]$ (A is a subset of the Euclidian space of dimension M) maps input $x\in R_+^N$ into subset $P(x)\subseteq R_+^N$ of output. The set $P(x)$ is called output set and it denotes the collection of all output vectors $y\in R_+^M$ that are obtainable from the input vector $x\in R_+^N$ (Neogi, 2004). The input correspondence $L:R_+^M\rightarrow 2^{R_+^N}$ maps the output $y\in R_+^M$ into subset $L(y)\subseteq R_+^N$ of inputs.

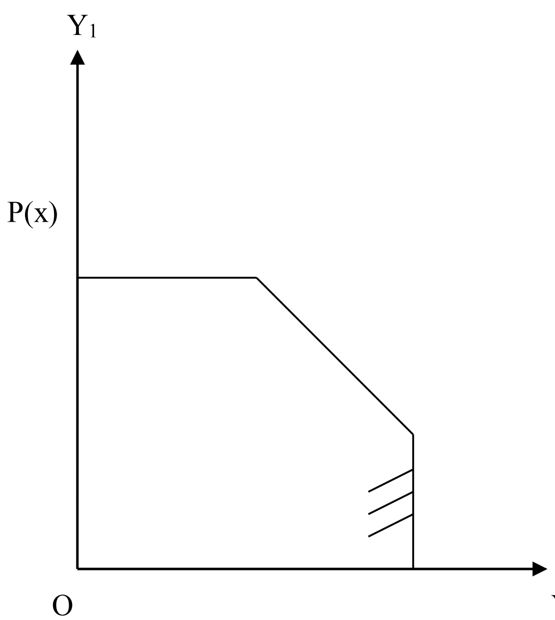


Figure 3.1.1

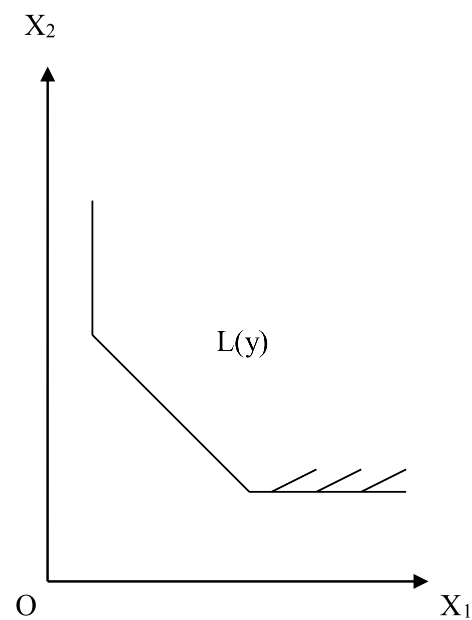


Figure 3.1.2

The input set $L(y)$ denotes the collection of all input vectors $x \in R_{+}^{N}$ that yields at least output vector $y \in R_{+}^{M}$. The input and output correspondence can be derived from one another by means of the relationships $L(y) = \{x : y \in P(x)\}$ and $P(x) = \{y : x \in L(y)\}$.

Now the graph of the production technology is the collection of all feasible input-output vectors, i.e.

$$GR = \{(x, y) \in R_{+}^{N+M} : y \in P(x), x \in R_{+}^{N}\} \text{ and}$$

$$GR = \{(x, y) \in R_{+}^{N+M} : x \in L(y), y \in R_{+}^{M}\}.$$

Thus the input-output correspondence can be derived from the graphs above (figure 3.1.1 and 3.1.2) as,

$$P(x) = \{y : (x, y) \in GR\} \text{ and}$$

$$L(y) = \{x : (x, y) \in GR\}.$$

Figure 3.1.3 models both inputs and output substitution in addition to modelling input-output transformation. The input set, the output set and the graph represents the technology in terms of input quantities and output quantities.

Let us introduce the concept of production frontier as a functional characteristic of the boundary of the graph of the production technology. The boundary of the graph represents the maximum possible output obtained from a given level of input or minimum input use for a given level of output. In a single output-multiple input case the production frontier can be defined as

$$\begin{aligned} f(x) &= \max\{y : y \in P(x)\} \\ &= \max\{y : x \in L(y)\} \end{aligned}$$

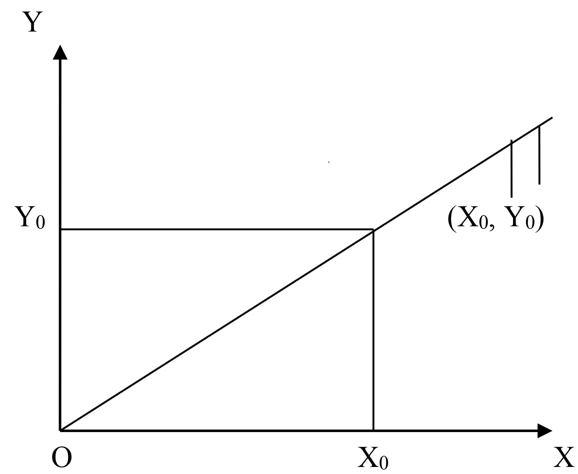


Figure 3.1.3

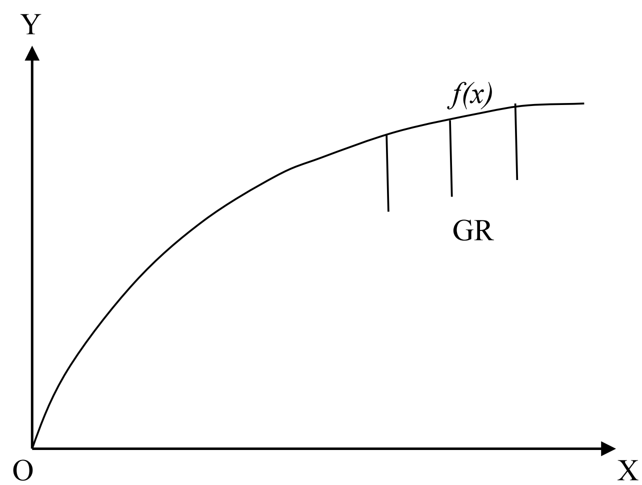


Figure 3.1.4

In figure 3.1.4 the production function $f(x)$ describes the maximum output that can be obtained with any given input vector. The different combinations of inputs and outputs fall on or below the production frontier. The basic idea of efficiency is to measure the distance of a particular combination of input and output of a production unit from the respective production frontier (Neogi, 2004).

There are two concepts of a distance function. The input distance function measures the maximum possible conservation of input to reach the boundary of production frontier. The concept can be illustrated in Figure 3.1.5 [(a) and (b)].

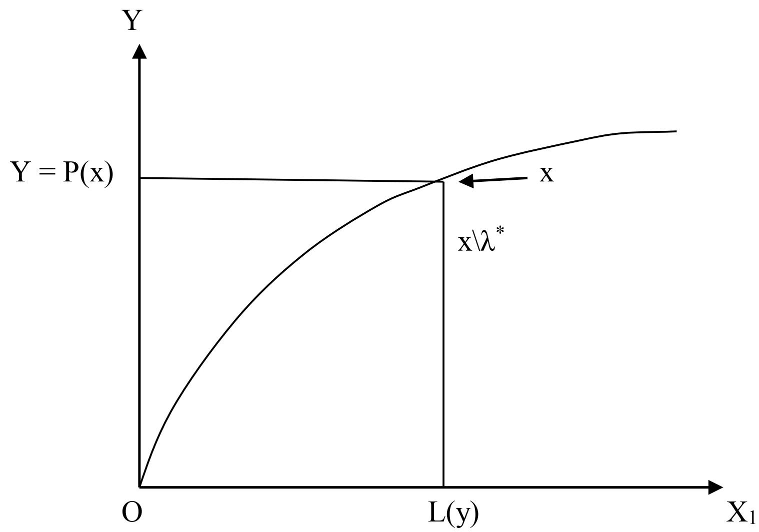


Figure 3.1.5 (a)

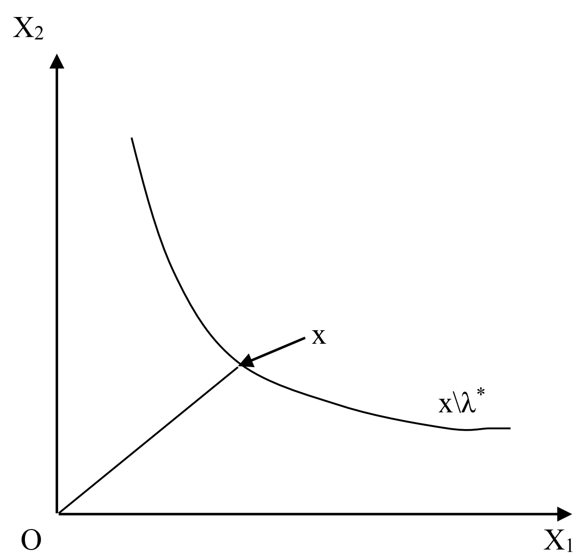
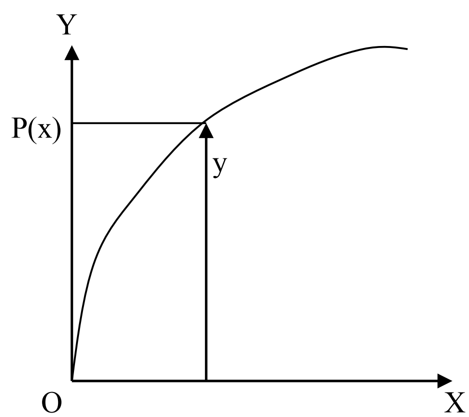


Figure 3.1.5 (b)

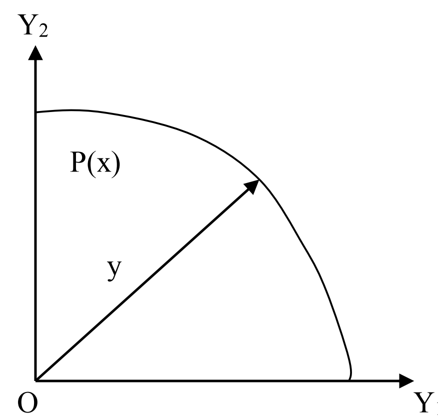
An input distance function can be defined as $D_i(y,x) = \max\{\lambda : x/\lambda \in L(y)\}$ where λ is the contraction factor by which inputs can be reduced to produce output *y*. Figure 3.1.5 is the graphical representation of the input distance function.

An output distance function can be defined as $D_0(y,x) = \min\{\mu : y/\mu \in P(x)\}$ where μ is the output expanding factor i.e., the proportion in which output can be maximized (with $\mu < 1$) with a given level of input. A graphical representation of the output distance function is given in figure 3.1.6 [(a) and (b)].



Input = 1; Output = 1

Figure 3.1.6 (a)



Output = 2

Figure 3.1.6 (b)

Efficiency measurement is based on the estimates of the best practice frontier production function which is a natural reference or basis of efficiency measure. Efficiency may be of three types: (i) technical, (ii) economic and (iii) scale.

Efficiency measure provides a description of the structure of an industry and is hence a very important step for identifying the causes of inefficiencies. Figure 3.1.7 (a) describes the concept of feasible production set which is the set of all input-output combinations which are feasible. The set consists of all points between the production frontier and the *X*- axis. The points along the production frontier line *OP* define the efficient subset of the feasible production set. If the firm operating at point A moves to point B, the firm can achieve output augmenting efficiency. Similarly if the firm moves from point A to point C, it will be technically efficient from the input saving perspective (measure). Point D in figure 3.1.7 (a) gives the technically optimal scale where output per unit of input is maximised. Figure 3.1.7 (b) represents the corresponding points of figure 3.1.7 (a) in an isoquant frame.

Now the output based measure of technical efficiency E_1 is computed by comparing an observed point of input requirement to produce output Y_a with the input requirement on the frontier production function corresponding to that level of output. In the input coefficient space this means comparing an observed input coefficient point with the point on the transformed isoquant of the frontier function corresponding to the observed output with observed factor proportions.

In figure 3.1.7 (b) this can be stated as $E_1 = \frac{OC}{OA}$. Another measure E_2 is obtained by comparing an observed point of input requirements for an observed output Y_a with the output Y_b obtained on the frontier production function at the same level of input. In 3.1.7 (b) this can be represented by the ratio $E_2 = \frac{OB}{OA}$.

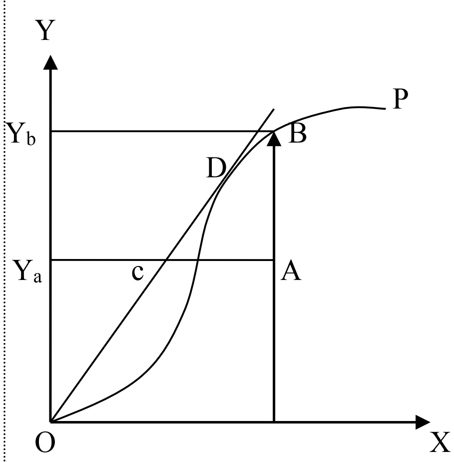


Figure 3.1.7 (a)

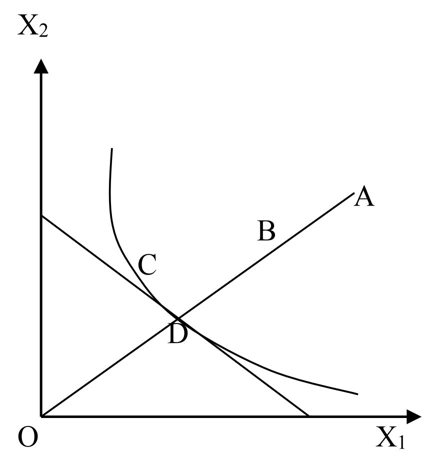


Figure 3.1.7 (b)

We now define the efficiencies in terms of distance functions. We consider the case of multiple inputs and single output. The input oriented measure of technical efficiency is given by the function $TE_i\,(y,x) = \min\,\{\lambda : y \le f(\lambda x)\}$ and the output oriented technical efficiency is measured as $TE_0\,(y,x) = \max\,\{\mu : y\mu \le f(\lambda x)\}$. Now the input oriented technical efficiency can be described as a measure of maximum radial contraction in *X* that enables to produce *Y* and $\lambda < 1$. Output oriented technical efficiency is the maximum radial expansion in *Y* for a given set of input *X*.

We must now introduce costs and input prices in measuring firm level efficiency. When input prices are introduced in explaining production technology it will be possible to measure the efficiency of units in terms of costs and allocation of inputs. A cost frontier is defined as the locus of minimum possible costs to produce a given level of output. A cost function is defined as $C\,(y,w) = \min\,\{W : x \in L(y)\}$, where $W = \sum w_i\, x_i$; w_i is the price of inputs.

The measure of cost efficiency is defined as $CE(y, x, w) = C(y, w)/W$. In other words it is the ratio of minimum possible costs to actual costs. In other words it is the ratio of minimum possible costs to actual costs (Neogi, 2004).

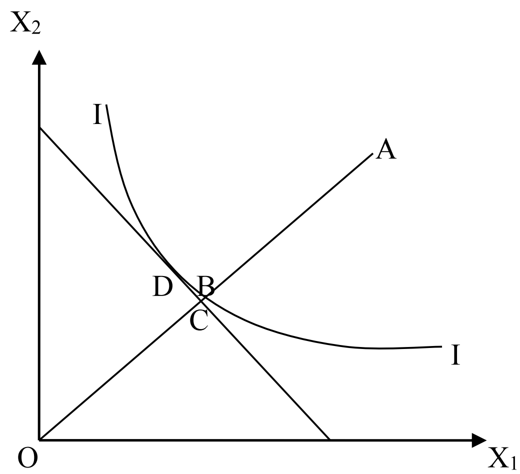


Figure 3.1.8

Let *A* be a set of inputs required to produce a given level of output as indicated by the isoquant *I I*. At point *A* the firm is neither technically efficient nor cost efficient. If the firm can move down to a point along the ray through the origin where it cuts the iso-cost line, the intersecting point will be cost efficient.

We define cost efficiency *CE* as the ratio of minimum costs of production with given input prices to observed cost. From figure 3.1.8 we can write $CE = \frac{OC}{OA}$. However all cost efficient points may not lie on the isoquant. That is, all cost efficient points are not technically efficient. For example, input combination at *C* in figure 3.1.8 is cost efficient but not technically efficient. On the other hand point *B* is technically efficient but not cost efficient. Allocative efficient point is a point which gives both technical and cost efficient combination of inputs. Point *D* in figure 3.1.8 is a point where the firm is technically efficient as well as cost efficient. Hence the point where allocative efficiency is attained must be a point of tangency between the iso-cost line and the isoquant. The measure of allocative efficiency is defined as $AE(y, x, w) = CE\ (y, x, w) / TE(y, x)$

The measure of input allocative efficiency is given by the ratio of cost efficiency to input oriented technical efficiency.

3.2. Formulation of the Stochastic Production Frontier

To begin with we assume that cross-sectional data on the quantities of m inputs used to produce a single output are available for each of *N* firms or producers in an industry.

A production frontier model can be written as

$$y_i = f(x_i ; \beta) .\ TE_i , \qquad (3.2.1)$$

Where y_i is the scalar of output of the i^{th} firm, i = 1, 2… N, x_i is a vector of m inputs used by firm i, $f(x_i ; \beta)$ is the production frontier, and β is a vector of technology parameters to be estimated. TE_i is the output oriented measure of technical efficiency of firm *i*. From the definition of technical efficiency developed in section 3.2.1, we can write

$$TE_i = \frac{y_i}{f(x_i ; \beta)} , \qquad (3.2.2)$$

which defines technical efficiency as the ratio of observed output to maximum feasible output. y_i achieves its maximum feasible value of $f(x_i ; \beta)$ if and only if $TE_i = 1$. Otherwise $TE_i <$ 1 provides a measure of the shortfall of observed output from maximum feasible output. In equation (3.2.1) the production frontier $f(x_i ; \beta)$ is a deterministic frontier. Consequently, in equation (3.2.2) the entire shortfall of observed output y_i from maximum feasible output $f(x_i ; \beta)$ is attributed to technical inefficiency. Such a formulation disregards the fact that output can be affected by random shocks that are beyond the control of firms. To incorporate firm specific random shocks into the analysis requires the specification of a stochastic production frontier. To do so we rewrite equation (3.2.1) as

$$y_i = f(x_i;\beta).\exp\{v_i\}.TE_i, \tag{3.2.3}$$

Where, $f(x_i;\beta).\exp\{v_i\}$ is the stochastic production frontier. The stochastic production frontier consists of two parts: a deterministic part $f(x_i;\beta)$ common to all producers and a producer specific part $\exp\{v_i\}$ which captures the effect of random shocks on each producer. Hence in case of a stochastic production frontier equation (3.2.2) becomes

$$TE_i = \frac{y_i}{f(x_i;\beta).\exp\{v_i\}}, \tag{3.2.4}$$

which defines technical efficiency as the ratio of observed output to maximum feasible output in an environment characterized by $\exp\{v_i\}$. Now y_i achieves its maximum possible value of $f(x_i;\beta).\exp\{v_i\}$ if and only if $TE_i = 1$. Otherwise $TE_i < 1$ provides a measure of the shortfall of observed output from maximum feasible output in an environment characterized by $\exp\{v_i\}$, which is allowed to vary across producers. Technical efficiency may be estimated using either the deterministic production frontier or the stochastic production frontier models as given in equations (3.2.1) and (3.2.3). In this study we prefer to use the stochastic production frontier model because the deterministic frontier ignores the effect of random shocks on the production process. The deterministic frontier runs the risk of improperly attributing unmodeled environmental variation to variation in technical efficiency (Kumbhakar, 2000).

3.3 The Basic Cross-Sectional Model and Method of Estimation

Aigner, Lovell and Schmidt (1977) (ALS) and Meeusen and van den Broeck (MB) (1977) simultaneously introduced the stochastic production frontier models. These models allow for technical inefficiency and also acknowledge the fact that random shocks outside the control of producers can affect output. The biggest advantage of the stochastic production is that the impact on output of shocks due to variation in labour and machinery performance, vagaries of the weather, and plain luck can in principle be separated from the contribution of variation in technical efficiency.

We assume that $f(x_i;\beta)$ takes the log-linear Cobb-Douglas form so that the stochastic production frontier can be written as

$$\ln y_i = \beta_0 + \sum_j \beta_j \ln x_{ji} + v_i - u_i \,, \tag{3.3.1}$$

Where, v_i is a two sided random statistical noise component and u_i is a non-negative ($u_i \geq 0$) technical inefficiency component of the error term. Since the error term in (3.3.1) has two components, the stochastic frontier model is often referred to as a composed error model. The noise component v_i is assumed to be independently and identically distributed (iid), is symmetric and is distributed independently of the one sided technical inefficiency component u_i. Thus the error term in (3.3.1) $\varepsilon_i = v_i - u_i$ is asymmetric since $u_i \geq 0$. Assuming that v_i and $u_i$are distributed independently of x_i, estimation of (3.3.1) by OLS provides consistent estimates of β_j s, but not of β_0, since $E(\varepsilon_i) = -E(u_i) \leq 0$. Moreover OLS does not provide producer specific technical efficiency.

However OLS provides a simple test for the presence of technical inefficiency in the given data. If $u_i = 0$, then $\varepsilon_i = v_i$, the error term is symmetric and the data do not support the presence of technical inefficiency. However if $u_i > 0$, then $\varepsilon_i = v_i - u_i$ is negatively skewed and there is evidence of technical inefficiency in the data. This implies that a test of the presence of inefficiency in the data can be directly based on the OLS residuals. Schmidt and Lin (1984) proposed the test statistic $b_1^{1/2} = \frac{m_3}{(m_2)^{3/2}}$, where m_2 and m_3 are the second and third sample moments of the OLS residuals. Since v_i is symmetrically distributed, m_3 is the third sample moment of u_i. Thus $m_3 < 0$ implies that the OLS residuals are negatively skewed and it suggests the presence of technical inefficiency. $m_3 > 0$ implies that the OLS residuals are positively skewed which is meaningless in this context. Hence positive skewness of OLS residuals indicates that the model is misspecified. Since the distribution of $b_1^{1/2}$ is not extensively published, Coelli (1995) proposed an alternative test statistic that is asymptotically distributed as $N(0,1)$. Negative skewness of OLS residuals occurs when $m_3 <$ 0, a test of hypothesis that $m_3 \geq 0$ is appropriate. Under the null hypothesis of zero skewness of errors in equation (3.3.1), the test statistic $\frac{m_3}{(6m_2^3 / N)^{1/2}}$ is asymptotically distributed as $N(0,1)$. The advantage of this test is that it is based on OLS residuals. The disadvantage is that it is based on asymptotic theory and especially in large scale manufacturing number of firms are small in a cross section. We shall consider hypothesis tests of the absence of technical inefficiency based on maximum likelihood estimators. In all the estimations presented in Chapter 4 we have routinely presented the value of the third central moment of OLS residuals.

In order to estimate the model in (3.3.1) ALS (1977) made specific distributional assumptions regarding the individual components of the composed error term ε_i. In particular the following assumptions are made.

(i) $v_i \sim iid\ N(0,\sigma_v^2)$

(ii) $u_i \sim iid\ N^+(0,\sigma_u^2)$, that is u_i is non-negative and distributed as half normal,

(iii) v_i and u_i are distributed independently of each other and of the regressors.

Assumption (i) is conventional and is maintained throughout our analysis. Assumption (ii) is based on the proposition that the modal value of the technical inefficiency term is zero. It is over simplistic and the distribution of the sum of v and u, under the distributional assumptions in (i) and (i) are easy to derive. The second part of assumption (iii) is a bit problematic as because if producers know something about their technical efficiency it can influence their choice of inputs. The density functions of v and u respectively are:

$$f(v)=\frac{1}{\sqrt{2\pi}\sigma_v}.\exp\left\{-\frac{v^2}{2\sigma_v^2}\right\} \tag{3.3.2}$$

and $$f(u)=\frac{2}{\sqrt{2\pi}\sigma_u}.\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\} \tag{3.3.3}$$

with $u_i \geq 0$, i.e. it is non-negative half normal. Given the independence assumption, the joint density function of v and u is the product of their individual density functions and hence

$$f(u,v)=\frac{2}{2\pi\sigma_u\sigma_v}.\exp\left\{-\frac{u^2}{2\sigma_u^2}-\frac{v^2}{2\sigma_v^2}\right\}. \tag{3.3.4}$$

Since $\varepsilon_i = v_i - u_i$, the joint density function of u and ε is

$$f(u,\varepsilon)=\frac{2}{2\pi\sigma_u\sigma_v}.\exp\left\{-\frac{u^2}{2\sigma_u^2}-\frac{(\varepsilon+u)^2}{2\sigma_v^2}\right\} \tag{3.3.5}$$

The marginal density function of ε is obtained by integrating u out of $f(u,\varepsilon)$, which gives

$$
\begin{aligned}
f(\varepsilon) &= \int_0^{\infty} f(u,\varepsilon)\,du \\
&= \frac{2}{\sqrt{2\pi}\sigma}.\left[1-\Phi\left(\frac{\varepsilon\lambda}{\sigma}\right)\right].\exp\left\{-\frac{\varepsilon^2}{2\sigma^2}\right\} \\
&= \frac{2}{\sigma}\phi\left(\frac{\varepsilon}{\sigma}\right).\Phi\left(-\frac{\varepsilon\lambda}{\sigma}\right)
\end{aligned}
\tag{3.3.6}
$$

where

$\sigma = (\sigma_u^2 + \sigma_v^2)^{1/2}, \lambda = \sigma_u / \sigma_v$ and $\phi(.)$ and $\Phi(.)$ are the standard normal density and cumulative distribution functions respectively. The reparameterisation from σ_u^2 and σ_v^2 to σ and λ is convenient as because λ provides an indication of the relative contribution of u and v in ε. As $\lambda \to 0$ either $\sigma_v^2 \to +\infty$ or $\sigma_u^2 \to 0$, and the symmetric error component dominates over the one sided error component in the determination of ε. As $\lambda \to \infty$ either $\sigma_u^2 \to +\infty$ or $\sigma_v^2 \to 0$ and the one sided error component dominated over the symmetric error component in determination of ε. In the first case we have the OLS production function model with no technical inefficiency, whereas in the second case we are back to a deterministic production frontier model with no noise.

The ALS (1977) stochastic production frontier model with normal-half normal composed error distribution contains basically two parameters, σ_u and σ_v or else σ and λ.

The distribution parameters σ and λ are to be estimated along with the technological parameters β. But before that we must test the hypothesis that $\lambda = 0$, where the test is based on the maximum likelihood estimate of λ. A likelihood ratio test may be conducted to test the hypothesis that $\lambda = 0$ but since the hypothesized value of λ lies on the boundary of the parameter space, it is difficult to interpret the test statistic. However Coelli (1995) has shown that in this case the appropriate one sided likelihood ratio test statistic is asymptotically distributed as a mixture of χ^2 distributions rather than as a single χ^2 distribution. The critical values of such a test are obtained in table Kodde and Palm (1986,). If the null hypothesis is true then the production function is equivalent to the traditional OLS average production function where firms are assumed to be fully technically efficient.

The marginal density function $f(\varepsilon)$ is asymmetrically distributed with mean and variance $E(\varepsilon) = -E(u) = -\sigma_u \sqrt{\frac{2}{\pi}}$ and $V(\varepsilon) = \frac{\pi - 2}{\pi} . \sigma_u^2 + \sigma_v^2$. ALS suggested [1 – *E*(*u*)] as an estimator of mean technical efficiency. But Lee and Tyler (1978) proposed

$$E(\exp\{-u\}) = 2[1 - \Phi(\sigma_u)].\exp\left\{\frac{\sigma_u^2}{2}\right\} \qquad (3.3.7)$$

which is preferred to [1 – *E*(*u*)] since [1 – *u*] includes only the first term in the power series expansion of $\exp\{-u\}$. Also unlike [1 – *E*(*u*)], $E(\exp\{-u\})$ is consistent with the definition of technical efficiency.

Using the marginal density function $f(\varepsilon)$ from (3.3.6) the log likelihood function for a sample of *N* firms in an industry is

$$\ln L = (constant) - N \ln \sigma + \sum_i \ln \Phi\left(-\frac{\varepsilon_i \lambda}{\sigma}\right) - \frac{1}{2\sigma^2} \sum_i \varepsilon_i^2 \qquad (3.3.8)$$

The log likelihood function in equation (3.3.8) can be maximized with respect to the parameters to obtain maximum likelihood estimates of all parameters of the model. These estimates are consistent as $N \to +\infty$.

Battese and Corra (1977) parameterization is more convenient from the estimation point of view. Letting $\gamma = \sigma_u^2 / \sigma^2$ we see that $\gamma \in [0,1]$. The log likelihood function with this reparameterisation is

$$\ln L = -\frac{N}{2}(\ln 2\pi + \ln \sigma^2) + \sum_i \ln \Phi(-z_i) - \frac{1}{2\sigma^2}\sum_i \varepsilon_i^2 \tag{3.3.9}$$

Where, $z_i = \left[\frac{\varepsilon_i}{\sigma}\right]\sqrt{\frac{\gamma}{1-\gamma}}$.

The next step is to estimate technical efficiency of each producer. We have estimates of $\varepsilon_i = v_i - u_i$, which contain information on u_i. The task is to extract the information that ε_i contains on u_i. A solution to the problem is obtained from the conditional distribution of u_i given ε_i, which contains whatever information ε_i has concerning u_i. Jondrow, Lovell, Materov and Schmidt (1982) showed that if $u_i \sim N^+(0, \sigma_u^2)$, the conditional distribution of u given ε_i is

$$\begin{aligned} f(u/\varepsilon) &= \frac{f(u,\varepsilon)}{f(\varepsilon)} \\ &= \frac{1}{\sqrt{2\pi}\sigma_*} \cdot \frac{\exp\left\{-\frac{(u-\mu_*)^2}{2\sigma_*^2}\right\}}{\left[1-\Phi\left(-\frac{\mu_*}{\sigma_*}\right)\right]} \end{aligned} \tag{3.3.10}$$

where $\mu_* = -\varepsilon \sigma_u^2 / \sigma^2$ and $\sigma_*^2 = \sigma_u^2 \sigma_v^2 / \sigma^2$. Since $f(u/\varepsilon)$ is distributed as $N^+(\mu_*, \sigma_*^2)$, either the mean or the mode of this distribution can serve as point estimator of u. The mean is given by

$$E(u_i / \varepsilon_i) = \mu_{*i} + \sigma_* \left[\frac{\phi(-\mu_{*i} / \sigma_*}{1 - \Phi(\mu_{*i} / \sigma_*)} \right]$$
$$= \sigma_* \left[\frac{\phi(\varepsilon_i \lambda / \sigma)}{1 - \Phi(\varepsilon_i \lambda / \sigma)} - \left(\frac{\varepsilon_i \lambda}{\sigma} \right) \right] \quad (3.3.11)$$

Once point estimates of u_i are obtained, estimates of firm specific technical efficiency can be computed from

$$TE_i = \exp\{-\hat{u}_i\} \quad (3.3.12)$$

Where, $\hat{u}_i$ is $E(u_i / \varepsilon_i)$. Battese and Coelli (1988) proposed the alternative point estimator for TE_i as,

$$TE_i = E(\exp\{-u_i\} / \varepsilon_i) = \left[\frac{1 - \Phi(\sigma_* - \mu_{*i} / \sigma_*)}{1 - \Phi(-\mu_{*i} / \sigma_*)} \right] . \exp\left\{ -\mu_{*i} + \frac{1}{2} . \sigma_*^2 \right\} \quad (3.3.13)$$

The point estimators in (3.3.12) and (3.3.13) can give different results since the two formulae are unidentical. We use the later in the present study. But it is to be noted that regardless of which estimator is used the estimates of technical efficiency are inconsistent simply because the variation associated with the distribution of (u_i / ε_i) is independent of *i*. However this is the best estimation strategy with cross sectional data.

3.4 The Normal-Truncated Normal Model

The normal–half normal model may be generalized by allowing *u* to follow a truncated normal distribution. This model was introduced by Stevenson (1980). The distributional assumption on *v* remains the same. Only the assumption on *u* is changed as u_i ~ $iid\ N^+(\mu, \sigma_u^2)$. The third assumption of the ALS model is also maintained. The truncated normal distribution assumed for *u* generalizes the one parameter half normal distribution by allowing the normal distribution which is truncated below at zero to have a non-zero mode. Thus this model contains an additional parameter μ to be estimated and hence provides a somewhat more flexible representation of the pattern of inefficiency in the data. The truncated normal density function for $u \geq 0$ is given by

$$f(u)=\frac{1}{\sqrt{2\pi}\sigma_u\,\Phi(-\mu/\sigma)}.\exp\left\{-\frac{(u-\mu)^2}{2\sigma_u^2}\right\} \qquad (3.4.1)$$

Here μ is the mode of the normal distribution which is truncated below at zero. If μ =0, then the density function collapses to the half normal density function. μ may be of either sign. The estimation strategy is the same as in ALS model but with minor changes in the parameterization. The log likelihood function for a sample of N firms is

$$\begin{aligned}\ln L = (cons\tan t) - N\ln\sigma \;-N\ln\Phi\left(-\frac{\mu}{\sigma_u}\right)\\ +\sum_i \ln\Phi\left(\frac{\mu}{\sigma\lambda}-\frac{\varepsilon_i\lambda}{\sigma}\right)-\frac{1}{2}\sum\left(\frac{\varepsilon_i+\mu}{\sigma}\right)^2\end{aligned} \qquad (3.4.2)$$

where $\sigma_u=\lambda\sigma/\sqrt{1+\lambda^2}$, $\sigma=(\sigma_u^2+\sigma_v^2)^{1/2}$ *and* $\lambda=\sigma_u/\sigma_v$. This log likelihood function can be maximized to obtain maximum likelihood estimators of all parameters in the model. It can be shown that the conditional distribution $f(u/\varepsilon)$ is distributed as $N^+(\tilde{\mu}_i,\sigma_*^2)$ where $\tilde{\mu}_i=(-\sigma_u^2\,\varepsilon_i+\mu\sigma_v^2)/\sigma^2$ and $\sigma_*^2=\sigma_u^2\,\sigma_v^2/\sigma^2$. Thus either the mean or the mode of $f(u/\varepsilon)$ may be used to estimate the technical efficiency of each firm. The mean of the conditional distribution of u is given by

$$E(u_i/\varepsilon_i)=\sigma_*\left[\frac{\tilde{\mu}_i}{\sigma_*}+\frac{\phi(\tilde{\mu}_i/\sigma_*)}{1-\Phi(\tilde{\mu}_i/\sigma_*)}\right] \qquad (3.4.3)$$

Point estimates of technical efficiency of each firm can be obtained by means of

$$TE_i = E(\exp\{-u_i\}/\varepsilon_i) = \left[\frac{1-\Phi[\sigma_* - (\tilde{\mu}_i/\sigma_*)]}{1-\Phi(-\tilde{\mu}_i/\sigma_*)}\right].\exp\left\{-\tilde{\mu}_i + \frac{1}{2}.\sigma_*^2\right\} \quad (3.4.4)$$

This produces unbiased but inconsistent estimates of technical efficiency (Kumbhakar and Lovell, 2000).

3.5 Panel Data Models with Time Invariant Technical Efficiency

Maximum likelihood estimation of a stochastic frontier production function model for panel data with time invariant technical efficiency is structurally similar to that applied to cross sectional data. We assume T time period observations on each of the N firms of our cross-section. Our production function model is of log linear Cobb-Douglas form as assumed in section 3.3.1 with the time subscripts attached appropriately.

$$\ln y_{it} = \beta_0 + \sum_j \beta_{jt} \ln x_{jit} + v_{it} - u_i \,, \quad (3.5.1)$$

We start by making the following distributional assumptions on the error components of the stochastic production frontier model as given in equation (3.5.1).

(iv) $v_{it} \sim iid\ N(0, \sigma_v^2)$

(v) $u_i \sim iid\ N^+(0, \sigma_u^2)$, that is u_i is non-negative and distributed as half normal,

(vi) v_{it} and u_i are distributed independently of each other and of the regressors.

These distributional assumptions are quite similar to those of the ALS model based on cross-sectional data. Now the noise term varies across firms as well as over time.

Pitt and Lee (1981) used these assumptions to estimate technical efficiency using panel data. The density function of $\boldsymbol{v} = (v_1, v_2, ..., v_T)'$, which is now time dependent is given by the following generalization of equation (3.5.2), where the firm subscripts are suppressed:

$$f(v) = \frac{1}{(2\pi)^{1/2}\sigma_v^T} . \exp\left\{-\frac{v'v}{2\sigma_v^2}\right\} \quad (3.5.2)$$

Given the independence assumption, the joint density function of u and v is

$$f(u, v) = \frac{2}{(2\pi)^{(T+2)/2}\sigma_u \sigma_v^T} . \exp\left\{-\frac{u^2}{2\sigma_u^2} - \frac{v'v}{2\sigma_v^2}\right\} \quad (3.5.3)$$

And the joint density function of u and $\varepsilon = (v_1 - u, v_2 - u, ..., v_T - u)'$ is

$$f(u, \varepsilon) = \frac{2}{(2\pi)^{(T+1)/2}\sigma_u \sigma_v^T} . \exp\left\{-\frac{(u-\mu_*)^2}{2\sigma_*^2} - \frac{\varepsilon'\varepsilon}{2\sigma_v^2} + \frac{\mu_*^{\ 2}}{2\sigma_*^2}\right\} \quad (3.5.4)$$

where

$$\mu_* = -\frac{T\sigma_u^2 \bar{\varepsilon}}{\sigma_v^2 + T\sigma_u^2}$$

$$\sigma_*^2 = \frac{\sigma_u^2 \sigma_v^2}{\sigma_v^2 + T\sigma_u^2}$$

$$\bar{\varepsilon} = \frac{1}{T}\sum_t \varepsilon_{it}$$

Thus the marginal density function of ε is obtained by integrating u out of $f(u, \varepsilon)$, which gives

$$\begin{aligned} f(\varepsilon) &= \int_0^\infty f(u, \varepsilon)\, du \\ &= \frac{2[1 - \Phi(-\mu_* / \sigma_*)]}{(2\pi)^{T/2}\sigma_v^{T+1}(\sigma_v^2 + T\sigma_u^2)^{1/2}} . \exp\left\{-\frac{\varepsilon'\varepsilon}{2\sigma_v^2} + \frac{\mu_*^{\ 2}}{2\sigma_*^2}\right\} \end{aligned} \quad (3.5.5)$$

The log likelihood function for a sample of N firms each observed over T time periods, is

$$\ln L = (constant) - \frac{N(T-1)}{2}\ln\sigma_v^2 - \frac{N}{2}\ln(\sigma_v^2 + T\sigma_u^2)$$
$$+\sum_i \ln\left[1-\Phi\left(\frac{\mu_{*i}}{\sigma_*}\right)\right] - \frac{\sum_i \varepsilon_i'\varepsilon_i}{2\sigma_v^2} + \frac{1}{2}\sum_i\left(\frac{\mu_{*i}}{\sigma_*}\right)^2 \quad (3.5.6)$$

This log likelihood function can be maximised with respect to the parameters to obtain the maximum likelihood estimates of $\beta, \sigma_v^2, and\ \sigma_u^2$.

The next step is to obtain estimates of firm specific time invariant technical efficiency. The conditional distribution of (u/ε) is

$$f(u/\varepsilon) = \frac{f(u,\varepsilon)}{f(\varepsilon)}$$
$$= \frac{1}{(2\pi)^{1/2}\sigma_*[1-\Phi(-\mu_*/\sigma_*)]}.\exp\left\{-\frac{(u-\mu_*)^2}{2\sigma_*^2}\right\} \quad (3.5.7)$$

This is the density function of a variable distributed as $N^+(\mu_*, \sigma_*^2)$. Either the mean or mode of this distribution can be used as a point estimator of technical efficiency. The mean is given by

$$E(u_i/\varepsilon_i) = \mu_{*i} + \sigma_*\left[\frac{\phi(-\mu_{*i}/\sigma_*)}{1-\Phi(\mu_{*i}/\sigma_*)}\right] \quad (3.5.8)$$

These estimates of u_i are consistent as $T \to \infty$. The values obtained from (3.5.7) can be substituted into $TE_i = \exp\{-\hat{u}_i\}$ to obtain firm specific estimates of time –invariant technical efficiency. An alternative estimator is provided by the minimum squared error predictor

$$E(\exp\{-u_i\}/\varepsilon_i) = \left[\frac{1-\Phi(\sigma_* - \mu_{*i}/\sigma_*)}{1-\Phi(-\mu_{*i}/\sigma_*)}\right].\exp\left\{-\mu_{*i} + \frac{1}{2}.\sigma_*^2\right\} \quad (3.5.9)$$

Pitt and Lee (1981) model shows the use of maximum likelihood techniques to obtain estimates of producer specific time invariant technical efficiency.

In case of panel data, the normal-truncated normal model was proposed by Kumbhakar (1987) and Battese and Coelli (1988). The log likelihood function for the normal –truncated normal model is given by

$$\ln L = (cons\tan t) - \frac{N(T-1)}{2}\ln\sigma_v^2 - \frac{N}{2}\ln(\sigma_v^2 + T\sigma_u^2) - N\ln\left[1-\Phi\left(-\frac{\mu}{\sigma_u}\right)\right]$$
$$+\sum_i \ln\left[1-\Phi\left(\frac{\tilde{\mu}_i}{\sigma_*}\right)\right] - \frac{\sum_i \varepsilon_i'\varepsilon_i}{2\sigma_v^2} - \frac{N}{2}\left(\frac{\mu}{\sigma_u}\right)^2 + \frac{1}{2}\sum_i\left(\frac{\tilde{\mu}_i}{\sigma_*}\right)^2 \qquad (3.5.10)$$

where $\tilde{\mu}_i = \dfrac{\mu\sigma_v^2 - T\bar{\varepsilon}\sigma_u^2}{\sigma_v^2 - T\sigma_u^2}$, σ_*^2 and $\bar{\varepsilon}$ are defined as before. $\tilde{\mu}_i = \mu_i^*$ if μ = 0. The log likelihood function can be maximized with respect to the parameters to obtain maximum likelihood estimates of all parameters. The conditional distribution (u/ε) is given by

$$f(u/\varepsilon) = \frac{f(u,\varepsilon)}{f(\varepsilon)}$$
$$= \frac{1}{(2\pi)^{1/2}\sigma_*[1-\Phi(-\tilde{\mu}/\sigma_*)]}.\exp\left\{-\frac{(u-\tilde{\mu})^2}{2\sigma_*^2}\right\} \qquad (3.5.11)$$

which is distributed as $N^+(\tilde{\mu},\sigma_*^2)$. Either the mean or the mode of the distribution can serve as a point estimate of firm specific technical efficiency. These are given by

$$E(u_i/\varepsilon_i) = \tilde{\mu}_i + \sigma_*\left[\frac{\phi(-\tilde{\mu}_i/\sigma_*)}{1-\Phi(\tilde{\mu}_i/\sigma_*)}\right] \qquad (3.5.12)$$

Producer specific estimates can be computed by using the minimum squared error predictor

$$E(\exp\{-u_i\}/\varepsilon_i) = \left[\frac{1-\Phi(\sigma_* - \tilde{\mu}_i/\sigma_*)}{1-\Phi(-\tilde{\mu}_i/\sigma_*)}\right].\exp\left\{-\tilde{\mu}_i + \frac{1}{2}.\sigma_*^2\right\} \qquad (3.5.13)$$

These point estimates can be inserted into $TE_i = \exp\{-\hat{u}_i\}$ to obtain firm specific estimates of time-invariant technical efficiency.

3.6 Panel Data Model with Time Varying Technical Efficiency

A stochastic production frontier model for panel data with time varying technical efficiency may be written as

$$\ln y_{it} = \beta_0 + \sum_j \beta_{jt} \ln x_{jit} + v_{it} - u_{it} , \qquad (3.6.1)$$

If the statistical independence assumption of the error components u and v are valid it is possible to use maximum likelihood technique to estimate the time-varying technical efficiency model. For an unbalanced panel data Battese and Coelli (1992) assumed that,

(i) $v_{it} \sim iid\ N(0, \sigma_v^2)$,

(ii) $u_{it} = \beta(t).u_i$,

(iii) $\beta(t) = \exp\{-\eta(t - T)\}$, and

(iv) $u_i \sim iid\ N^+(\mu, \sigma_u^2)$ i.e., u follows a truncated normal distribution.

The function $\beta(t)$ satisfies the properties: (a) $\beta(t)$, (b) $\beta(t)$ decreases at an increasing rate if $\eta > 0$, increases at an increasing rate if $\eta < 0$, or remains constant if $\eta = 0$. Battese and Coelli (1992) used maximum likelihood estimation method to find estimates of all parameters of the model. Based on the distributional assumptions they showed that $u_i / \varepsilon_i \sim iid\ N^+(\mu_{**}, \sigma_*^2)$, where $\varepsilon_i = v_i - \beta.u_i$ or $\varepsilon_i = \ln y_i - \beta_o - \sum_j \beta_j \ln x_{ji}$ with

$$\mu_{**i} = \frac{\mu\sigma_v^2 - \beta'\varepsilon_i\sigma_u^2}{\sigma_v^2 + \beta'\beta\sigma_u^2}$$

$$\sigma_*^2 = \frac{\sigma_u^2\sigma_v^2}{\sigma_v^2 + \beta'\beta\sigma_u^2}$$

$$\beta' = [\beta(1)\,\beta(2)........\beta(T)]$$

If technical efficiency is time invariant then η = 0, which implies $\beta(t) = 1$ and $\beta'\beta = T$ so that the expressions for μ_{**i} and σ_*^2 collapse to their time invariant versions given below equation (3.5.4).

However for the purpose of estimation the Battese and Corra (1977) parameterization is very useful. They suggested the parameterization of the likelihood function in terms of $\sigma^2 = \sigma_v^2 + \sigma_u^2$ and $\gamma = \sigma_u^2 / (\sigma_v^2 + \sigma_u^2)$. This is done with the calculation of the maximum likelihood estimates in mind. The parameter, γ, must lie between 0 and 1 and thus this range can be searched to provide a good starting value for use in an iterative maximization process such as the Davidson-Fletcher-Powell (DFP) algorithm. The log likelihood function, in terms of this parameterization, is given in Battese and Coelli (1992). The minimum mean squared error predictor is given by

$$E(\exp\{-u_{it}\}/\varepsilon_i) = E(\exp\{\beta(t).u_i\}/\varepsilon_i)$$
$$= \left[\frac{1-\Phi(\beta(t)\sigma_* - \mu_{*i}/\sigma_*)}{1-\Phi(-\mu_{*i}/\sigma_*)}\right].\exp\left\{-\beta(t)\mu_{*i} + \frac{1}{2}.\beta(t)^2\,\sigma_*^2\right\} \qquad (3.6.2)$$

For testing of several restrictive forms the Battese and Coelli (1992) model is extremely helpful. We present a list of models nested in the Battese and Coelli (1992) model as follows:

Model I: Battese and Coelli (1992) model with balanced panel data (used in our study).

Model II: Setting $\gamma = 0$ gives the traditional OLS average production function model where firms are assumed to be fully technically efficient.

Model III: Setting η =0 and restricting the formulation to a full (balanced) panel of data gives the time-invariant model set out in Battese and Coelli (1988) or Kumbhakar (1987).

Model IV: The additional restriction (over model III) of μ equal to zero reduces the model to the normal-half normal model in Pitt and Lee (1981).

Model IV: Adding another restriction of T=1 to returns to the original cross-sectional, normal half-normal formulation of Aigner, Lovell and Schmidt (1977).

Model V: All these restrictions excepting μ= 0, gives the cross sectional normal truncated normal model suggested by Stevenson (1980).

The null hypothesis of no technical inefficiency (Model II) can be tested by applying the Likelihood Ratio Test. The likelihood ratio test is based on the likelihood ratio statistic (*LR*) defined as,

$$LR = - 2ln[L(H_0) / L(H_A)],$$

Where, $L(H_0)$ and $L(H_A)$ are the values of the likelihood function (optimum) under the null and alternative hypotheses respectively. The *LR* statistic is approximately distributed as a mixed χ^2 distribution as because γ= 0 is a value on the boundary of the parameter space of γ (Coelli, 1995). Analogously other tests of restrictions or set of restrictions of the full stochastic frontier model may be conducted using the usual likelihood ratio test (the χ^2 test).

3.7 Cobb-Douglas Cost Function

In order to estimate Cobb-Douglas cost function a few simplifying assumptions are made. Admittedly credible information on price of land is difficult to obtain and ascertain. This is especially true for land meant for tea plantations. Thus if the factor land (or alternatively area under cultivation) is eliminated from a Cobb-Douglas production function, it would consequently be eliminated from the Cobb-Douglas cost function simply because it is derived from the Cobb-Douglas production function. Thus, for the present analysis the production function is taken in per hectare form, so that the variable factor land or cultivated area is eliminated from the inputs. Hence, the study considers all the other factors in per hectare form, namely, wage cost per hectare, irrigation cost per hectare, and pesticides-fertilizer cost per hectare. So, expressing the Cobb-Douglas production function with three endogenous inputs in per hectare form, the corresponding Cobb-Douglas cost function may be written as,

$$C_i = \mu . Q^{\frac{1}{v}} . w^{\frac{\alpha_1}{v}} . i^{\frac{\alpha_2}{v}} . p^{\frac{\alpha_3}{v}} . \qquad \text{................. (3.7.1)}$$

Here*w, i* and *p* are real prices of labour per hectare, irrigation per hectare and pesticides and fertilizers per hectare respectively, μ is a constant and *C* is obviously total cost of plantation or cultivation expressed in per hectare form. Q is clearly output per hectare. Economic interpretation of the partial output elasticity parameters of the Cobb-Douglas production function is important. $\alpha_1$is the partial elasticity of output per hectare with respect to labour use per hectare. Similar explanations in case of irrigation, pesticides and fertilisers may be provided for the elasticity parameters α_2 and α_3 respectively.

The original production function parameters are called the structural form parameters. They are the time varying technological coefficient *A(t)*, and the exponents of the inputs expressed in per hectare form. On the other hand, the parameters of the Cobb-Douglas cost functionμ, $1/v$, α_1/v, α_2/v and α_3/v are actually reduced form parameters where $\alpha_1 + \alpha_2 + \alpha_3 = v$.

It is observed that the number of reduced form parameters is greater than the number structural form parameters and thus the model is over identified. This is usually true for any Cobb-Douglas cost function which is derived from the production function using the conditional factor demand functions found from the first order conditions of cost minimisation (subject to output constraint). Nerlove (1963) devised a method of imposing parameter restrictions in such a way that the number of structural form parameters are exactly equal to the number of reduced form parameters. Following Nerlove, the following restrictions on the parameters are imposed.

$$\text{If } \alpha_1 + \alpha_2 + \alpha_3 = \nu,\ \alpha_1 = \nu - \alpha_2 - \alpha_3 \qquad \text{--------(3.7.2)}$$

Replacing equation3.7.2 in equation3.7.1, we get

$$C_i = \mu . Q^{\frac{1}{\nu}} . w^{\frac{\nu - \alpha_2 - \alpha_3}{\nu}} . i^{\frac{\alpha_2}{\nu}} . p^{\frac{\alpha_3}{\nu}} .$$

which yields

$$\frac{C_i}{W_i} = \mu . Q_i^{\frac{1}{\nu}} . \left[\frac{I_i}{W_i}\right]^{\frac{\alpha_2}{\nu}} . \left[\frac{P_i}{W_i}\right]^{\frac{\alpha_3}{\nu}}$$

Finally, taking log on both sides we have

$$\log\left[\frac{C_i}{w_i}\right] = \left[\log \mu + \frac{1}{\nu}\log Q_i + \frac{\alpha_2}{\nu}\log\left(\frac{i_i}{w_i}\right) + \frac{\alpha_3}{\nu}\log\left(\frac{p_i}{w_i}\right)\right]$$

$$or \qquad \log\left[\frac{c_i}{w_i}\right] = \left[\log \mu + \frac{\alpha_2}{\nu}\log\left(\frac{i_i}{w_i}\right) + \frac{\alpha_3}{\nu}\log\left(\frac{p_i}{w_i}\right)\right] + \frac{1}{\nu}\log Q_i$$

$$or \qquad \log\left(\frac{c_i}{w_i}\right) = \Omega + \frac{1}{\nu}\log Q_i + u_i \qquad \text{-----------------(3.7.3)}$$

The study makes an over simplistic assumption that relative price of inputs in tea plantations stay more or less unchanged in the medium to long run. In other words the ratios *i/w*, and *p/w* are assumed to be constant over the ten year period 2001-10. Hence the sum of the first three terms on the right hand side of log-linear expression above may be assumed to be a pure constant Ω.

Conceptually price of irrigation is difficult to imagine and measure. However wage rate is measureable but difficult to estimate at the garden level due to several reasons. First, most gardens do not disclose the exact number of permanent and casual workers. Permanent staffs are paid a much higher wage rate compared to the casual staff. For most gardens the ratio of casual to permanent workers is around 70 is to 30. This figure may of course vary slightly across gardens. Thus a weighted average wage rate calculation depends exactly on the ratio or proportion which is unknown on most occasions.

Second casual workers clearly disclose that they do not receive the wage rate according to the balance sheet declared rate. They rather receive around 50 per cent of the balance sheet declared wage rate. This further complicates the computation or estimation of an average wage rate at the garden level on an annual basis over the study period. The most important issue with regard to estimation however is that the true proportion of casual workers is never disclosed. The present study bypasses these controversial methodological issues and assumes that workers are paid on an average, the company declared wage rate as recorded in their annual balance sheets. More over the relative factor price ratios i/w, and p/w are constant.

3.8 Cross-Sectional Cost Frontier Model

A cost frontier can be treated as a single-equation model, just as a production frontier. In this case it is possible to obtain estimates of the parameters describing the structure of the cost frontier, as well as producer-specific estimates of cost efficiency simultaneously. However, if input quantity data or input cost share data are available and if Shephard's lemma is applied, a cost frontier can be treated under a simultaneous-equation framework. It is also possible to obtain producer-specific estimates of the magnitude and cost of technical efficiency and the magnitude and cost of input allocative efficiency. However, moving from a single-equation model to a simultaneous-equation model requires more data and involves a more complicated estimation system, but it offers the possibility of gaining more insight into the nature of cost efficiency. Despite its obvious limitations the single equation cost frontier model is employed in the present study in order to estimate firm specific cost efficiency as well as the maximum likelihood estimates of parameters of the cost frontier. The single-equation cost frontier model is presented in Section 3.8.1.

3.8.1 Single-Equation Cost Frontier Models

To start with it is assumed that cross-sectional data on the prices of inputs employed, the quantities of outputs produced, and total expenditure are available for each of *i* producers. The analysis is based on a cost frontier, which can be expressed as

$$E_i \geq c(y_i, w_i; \beta), \quad i = 1 \; \ldots . l \ldots \ldots \ldots (3.8.1.1)$$

Where $E_i = w_i^T x_i = \sum_n w_{ni} x_{ni}$ is the expenditure incurred by producer i, $y_i = (y_{li}, \ldots \ldots y_{Mi}) \geq 0$ is a vector of outputs produced by producer *i*. $w_i = (w_{li}, \ldots \ldots w_{Ni}) > 0$is a vector of input prices faced by producer i, $c(y_i, w_i: \beta)$ is the cost frontier common to all producer, and β is a vector of technology parameters to be estimated. It is to be noticed that the input vector x_i used by producer i is not necessarily observed. If it is not observed, cost efficiency cannot be decomposed into the cost of input-oriented technical inefficiency and the cost of input allocative inefficiency. If it is observed, the decomposition can be achieved.

Cost efficiency of the ith producer may be written as,

$$CE_i = \frac{c(y_i, w_i: \beta)}{E_i} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.8.1.2)$$

which defines cost efficiency as the ratio of minimum feasible cost to observed expenditure. Since $E_i \geq c(y_i, w_i; \beta)$, it follows that $CE_i \leq 1. CE_i = 1$if, and only if ,$x_{ni} = x_{ni}(y_i, w_i; \beta) \forall_n$ so that $E_i = \sum_n w_{ni} x_{ni}(y_i, w_i: \beta)$attains its minimum feasible value of $c(y_i, w_i: \beta)$. Otherwise $CE_i < 1$ provides a measure of the ratio of minimum cost to observed expenditure.

In equation 3.8.1.1 the cost frontier $c(y_i, w_i: \beta)$ is deterministic, and so in equation 3.8.1.2 the entire excess of observed expenditure over minimum feasible cost is attributed to cost inefficiency. Such a formulation ignores the fact that expenditure may be affected by random shocks not under the control of producers. A stochastic cost frontier can be written as

$$E_i \geq c(y_i, w_i; \beta).exp\{v_i\}\text{..................................}(3.8.1.3)$$

where, $[c(y_i, w_i; \beta).exp\{v_i\}]$ is the stochastic cost frontier. The stochastic cost frontier consists of two parts, a deterministic part $c(y_i, w_i; \beta)$that is common to all producers, and a producer specific random part $exp\{v_i\}$ which captures the effects of random shocks on each producer. If the cost frontier is specified as being stochastic, the appropriate measure of cost efficiency becomes

$$CE_i = \frac{c(y_i, w_i{:}\beta).exp\{v_i\}}{E_i} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3.8.1.4).$$

This defines cost efficiency as the ratio of minimum cost attainable in an environment characterized by $exp\{v_i\}$ to observed expenditure. $CE_i \leq 1$, $CE_i = 1$ if and only if, $E_i = c(y_i, w_i; \beta).exp.\{v_i\}$. Otherwise$CE_i < 1$ provides a measure of the ratio of the minimum feasible cost to observed expenditure. The estimation of cost efficiency can be based on either equation 3.8.1.1 or equation 3.8.1.3.

3.8.2 The single output Cobb-Douglas Cost Frontier

If we assume that the deterministic kernel $c(y_i, w_i; \beta)$ of the single output cost frontier takes the log linear Cobb-Douglas functional form, then the stochastic cost frontier model gives in equation 3.8.1.3 can be written as

$$lnE_i \geq \beta_0 + \beta_v \ln y_i + \sum_n \beta_n lnw_{ni} + \mathrm{v}_i$$

$$=\beta_0 + \beta_v \ln y_i + \sum_n^n \beta_0 + \beta_v \ln y_i + \mathrm{v}_i + u_i\text{.....................}(3.8.2.1)$$

Where v_i is the two sided random noise component, and u is the nonnegative cost inefficiency component, of the composed error term $\varepsilon_i = v_i + u_i$. Since a cost frontier must be linearly homogeneous in input prices, $c(y_i, \lambda w_i; \beta) = \lambda c(y_i, w_i; \beta), \lambda > 0,$ and either the parameter restriction $\beta_k = 1 - \sum_{n \neq k} \beta_n$must be imposed prior to estimation, or equation 3.8.2.1 must be reformulated as

$$ln\left[\frac{E_i}{w_{ki}}\right] = \beta_0 + \beta_v \ln y_i + \sum_{n \neq k} \beta_n \, ln\left[\frac{w_{ni}}{w_{ki}}\right] + v_i + u_i. \text{ ....(3.8.2.2)}$$

Using equation 3.8.1.4, a measure of cost efficiency is provided by

$$CE_i = \exp\{-u_i\}. \text{ ....(3.8.2.3)}$$

In both formulations of the stochastic cost frontier, the error term$\varepsilon_i = v_i + u_i$ is asymmetric, being positively skewed since $u_i \geq 0.$ apart from the homogeneity restriction on the $\beta_n s$ and the direction of the skewness of the error term, the stochastic cost frontier model given by equation 3.8.2.1 and 3.8.2.2 is structurally indistinguishable from the stochastic production frontier model given by equation 3.3.1. Thus, apart from some sign changes, the entire analysis of section 3.2.2is applicable to the estimation of stochastic cost frontier. If maximum likelihood techniques are employed to obtain estimates of β and the parameters of the two error components, the same distributional assumptions can be made for the error components in equation 3.8.2.1or equation 3.8.2.2. It is also possible to use method of moments estimation techniques to obtain estimates of β and the parameters of the two error components. In either case the JLMS decomposition can be used to separate noise from cost inefficiency in the residuals. The estimated cost inefficiency component can then be substituted in equation 3.8.2.3 to obtain producer-specific estimates of cost efficiency.

We now illustrate the use of maximum likelihood techniques to estimate the stochastic Cobb-Douglas cost frontier given in equation 3.8.2.2. We make the following distributional assumptions:

(i) $v_i \sim iid\ N(0, \sigma_v^2)$.

(ii) $u_i \sim iid\ N(0, \sigma_u^2)$.

(iii) v_i and u_i are distributed independently of each other, and of the regressors.

The density function of $u \geq 0$ is given in equation 3.3.3. The density function of v is given in equation 3.3.2. The marginal density function of $\epsilon = v + u$ is

$$f(\epsilon) = \int_0^{\propto} f(u, \epsilon) du$$

$$= \int_0^{\propto} \frac{2}{2\pi\sigma_u\sigma_v} . exp\left\{\frac{-u^2}{2\sigma_u^2} - \frac{(\epsilon-u)^2}{2\sigma_v^2}\right\} du$$

$$= \frac{2}{\sqrt{2\pi}\sigma} . \left[1 - \Phi\left(\frac{-\varepsilon\lambda}{\sigma}\right)\right] . exp\left\{-\frac{\varepsilon^2}{2\sigma^2}\right\}$$

$$= \frac{2}{\sigma} . \Phi\left(\frac{\varepsilon}{\sigma}\right) \Phi\left(\frac{\varepsilon\lambda}{\sigma}\right). \ldots\ldots\ldots\ldots\ldots(3.8.2.4)$$

Where, $\sigma = (\sigma_u^2 + \sigma_v^2)^{1/2}, \lambda = \frac{\sigma_u}{\sigma_v}$, and Φ(.) and Φ(.) are the standard normal cumulative distribution and density functions. As $\lambda \to 0$ either $\sigma_v^2 \to +\propto$or $\sigma_u^2 \to 0$, and the symmetric error component in the determination of ε. As $\lambda \to +\propto$ either $\sigma_u^2 \to +\propto$ or $\sigma_v^2 \to 0$ and the one sided error component dominates the symmetric error component in the determination of ε. In the former case the stochastic cost frontier model collapses to an OLS cost function model with no variation in cost efficiency, whereas in the latter case the model collapses to a deterministic cost frontier model with no noise. As in the estimation of technical efficiency, it is possible to conduct a likelihood ratio test of the hypothesis that $\lambda = o$.

The marginal density function $f(\varepsilon)$ is asymmetrically distributed, with mean and variance

$$E(\varepsilon) = E(u) = \sigma_u \sqrt{\frac{2}{\pi}},$$

$$V(\varepsilon) = \frac{\pi - 2}{\pi} \sigma_u^2 + \sigma_v^2 \text{ ....(3.8.2.5)}$$

Using equation 3.8.2.5, the log likelihood function for a sample of I producers is

$$\ln L = constant - I \ln \sigma + \sum_i \ln \Phi\left(\frac{\varepsilon_i \lambda}{\sigma}\right) - \frac{1}{2\sigma^2} \sum_i \varepsilon_i^2 \text{ ....(3.8.2.6)}$$

The log likelihood function can be maximized with respect to the parameters to obtain maximum likelihood estimates of all parameters.

The next step is to obtain the estimates of the cost efficiency of each producer. We have estimates of $\varepsilon_i = v_i + u_i$, which obviously contain information of u_i. If$\varepsilon_i < 0$, chances are that u_i is not large (since $E(v_i) = 0$), which suggest that this producer is relatively cost inefficient. The problem is to extract the information that ε_i contains u_i. A solution to the problem is obtained from the conditional distribution of u_i given ε_i , which contains whatever information ε_i contains concerning u_i. Adapting the JLMS (1982) procedure to the estimation of cost efficiency when$u_i \sim N^+(0, \sigma_u^2)$, the conditional distribution of u given ε is

$$f(u|\varepsilon) = \frac{f(u, \varepsilon)}{f(\varepsilon)}$$

$$= \frac{1}{\sqrt{2\pi}\sigma_*} . exp\left\{-\frac{(\mu - \mu_*)^2}{2\sigma_*^2}\right\} / \left[1 - \Phi\left(\frac{-\mu_*}{\sigma_*}\right)\right] \text{ ....(3.8.2.7)}$$

Where, $\mu_* = \varepsilon\sigma_u^2 / \sigma^2$ and $\sigma_*^2 = \sigma_u^2\sigma_v^2 / \sigma^2$. Since $f(u|\varepsilon)$ is distributed as $N^*(\mu_* \sigma_*^2)$ either the mean or the mode of this distribution can serve as a point estimator for u_i. given are by

$$E(u_i|\varepsilon_i = \mu_{*i} + \sigma_* \left[\frac{\Phi(-\frac{\mu_{*i}}{\sigma_*})}{1 - \Phi(-\frac{\mu_{*i}}{\sigma_*})}\right]$$

$$= \sigma_* \left[\frac{\Phi(\frac{\varepsilon_i\lambda}{\sigma})}{1-\Phi(\frac{-\varepsilon_i\lambda}{\sigma})} + \left(\frac{\varepsilon_i\lambda}{\sigma}\right)\right] \ldots\ldots\ldots\ldots(3.8.2.8)$$

And

$$M(u_i|\varepsilon_i) = \begin{cases} \varepsilon_i \left(\frac{\sigma_u^2}{\sigma^2}\right) & if\ \varepsilon_i \geq 0. \\ 0 & otherwise, \end{cases} \ldots\ldots\ldots\ldots\ldots\ldots(3.8.2.9)$$

respectively. Once point estimates of u_i are obtained, estimates of the cost efficiency of each producer can be obtained by substituting either $E(u_i|\varepsilon_i)$ or $M(u_i|\varepsilon_i)$ into equation 3.8.2.3. It is also possible to adapt the Battese and Coelli (1988) point estimator

$$CE_i = E(exp\{-u_i\}|\varepsilon_i) = \left[\frac{1-\Phi(\sigma_* - \frac{\mu_{*i}}{\sigma_*})}{1-\Phi(-\frac{\mu_{*i}}{\sigma_*}}\right].\, exp\left\{-\mu_{*i} + \frac{1}{2}\sigma_*^2\right\} \ldots\ldots(3.8.2.10)$$

The present study uses (3.8.2.10) to find point estimates of garden specific estimates of cost efficiency. The time varying cost efficiency model is not employed. In a nutshell while the production frontier specifies the error term as $(v_i - u_i)$ where v is the symmetric random noise component and u is the one sided inefficiency random variable, the cost frontier simply specifies the composed error as $(v_i + u_i)$ with the random variables remaining the same. This simple substitution changes a production frontier into a cost frontier. The quantity $u_i$now indicates the extent by which the tea garden operates above the cost frontier. Assuming allocative efficiency $u_i$is closely related to the cost of technical inefficiency [Coelli, 1996]. However if allocative efficiency is not assumed the interpretation of $u_i$in a cost function is not very clear and both technical and allocative inefficiencies may be simultaneously involved. Hence following Coelli (1996) the present study refers efficiencies measured with respect to the cost frontier as simply cost efficiency. The cost frontier employed in the present study is similar to the one proposed by Schmidt and Lovell (1979). The log-likelihood function of the cost frontier is the same as that of the production frontier with the exception of a few changes of signs.

3.9 Measurement of TFPG

The study adopts four different approaches to measure total factor productivity growth in the tea industry in Assam.

(1) Growth Accounting: The **Solow Divisia Index, Tornqvist Divisia Index,** and **Kendrick Index** are used to measure total factor productivity growth**.**

(2) Trans-Log Measure of Technical Change: The study estimates **Production** and **Cost Functions** to compute the rate of technical progress and bias with respect to inputs**.**

(3) Nonparametric Approach: **Malmquist TFP Index** is used to a non-parametric estimate of TFPG.

3.9.1 Methodological Issues

Total factor productivity indices themselves fall into two separate categories: (a) Arithmetic TFP indices [Abramovitz (1956); Kendrick (1961)]; (b) Geometric or Divisia TFP indices [Solow (1957); Jorgenson and Griliches (1967)] depending upon their definitions of I_t. The most important widely used variant of arithmetic indices is Kendrick index. Kendrick index (1961) of TFP is based on a linear production function which assumes infinite elasticity of substitution between factors of production. The Kendrick index is defined as:

$$P_t = \frac{Q_t}{\sum W_{i,0} . X_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.9.1.1)$$

Where, $W_{i,0}$ refers to the reward of the input i in the base year.

In order to compute the Geometric or Divisia indices of total factor productivity, we shall proceed as follows. Given the production function

$$Y = F(X_1, X_2, \ldots, X_k, t) \text{ ....(3.9.1.2)}$$

Under constant returns to scale, the construction of the divisia or the geometric index of total factor productivity that belongs to the growth accounting approach for measuring productivity is based on the following formula

$$DI = \frac{Y_t}{Y_0} \exp\left[-\sum_{i=1}^{k} \int_0^t Sh_i \frac{\dot{X}_i}{X_i} \right] \qquad \text{.....................(3.9.1.3)}$$

Where, Y is output, X's are inputs, t is time and Sh is the share of input in the value of output. This type of index was used by Abramowitz (1956), Solow (1956), and Jorgenson and Grilliches (1967) in their empirical studies. The logical foundation of this index was developed and enriched by Richter (1966), Gorman (1970), Hillinger (1970) and Hulten (1973).

Based on the production function (3.9.1.2), the total differential is

$$dY = F_1 dX_1 + F_2 dX_2 + \cdots + F_k dX_k + F_t dt$$

Or, $\frac{dY}{dt} = F_1 \frac{dX_1}{dt} + F_2 \frac{dX_2}{dt} + \cdots + F_k \frac{dX_k}{dt} + F_t$

Or, $$\frac{1}{Y}\frac{dY}{dt} = \frac{X_1 F_1}{Y}\frac{1}{X_1}\frac{dX_1}{dt} + \frac{X_2 F_2}{Y}\frac{1}{X_2}\frac{dX_2}{dt} + \cdots + \frac{X_k F_k}{Y}\frac{1}{X_k}\frac{dX_k}{dt} + \frac{F_t}{Y}$$

Or, $$\frac{F_t}{Y} = \frac{1}{Y}\frac{dY}{dt} - \sum_{i=1}^{k}\left[\left(\frac{X_i F_i}{Y} \right) \left(\frac{1}{X_i}\frac{dX_i}{dt} \right) \right]$$

Thus, the divisia index is given as

$$DI = \frac{\dot{Y}}{Y} - \sum_{i=1}^{k} Sh_i \frac{\dot{X}_i}{X_i} \text{ .... (3.9.1.4)}$$

where , $Sh_i = \frac{\partial \ln Y}{\partial \ln X_i} \approx \frac{X_i F_i}{Y}$ and $\sum_{i=1}^{k} Sh_i = 1$

The divisia index (3.8.1.4) that shows the rate of technical change is defined as the difference between the rate of growth of output and the weighted average of rates of growth of inputs, the weights being the shares of inputs in the value of output. For the economic time series data, Solow (1957) computed the divisia index by using the formula

$$DI_t = \left(\frac{\dot{Y}}{Y} - \frac{\dot{X}_k}{X_k} \right) - \sum_{i=1}^{k-1} Sh_i \left(\frac{\dot{X}_i}{X_i} - \frac{\dot{X}_k}{X_k} \right) = \left(\frac{\Delta Y}{Y} - \frac{\Delta X_k}{X_k} \right) - \sum_{i=1}^{k-1} Sh_i \left(\frac{\Delta X_i}{X_i} - \frac{\Delta X_k}{X_k} \right) \text{(3.9.1.5)}$$

Equation (3.9.1.5) gives the Solow residual measure of total factor productivity growth. For the present study where we have only two inputs, namely, capital (*K*) and labour (*L*), Solow residual for annual time series data, is

$$DI_t = \left(\frac{\Delta Y}{Y} - \frac{\Delta L}{L} \right) - (1 - Sh_L) \left(\frac{\Delta K}{K} - \frac{\Delta L}{L} \right) \text{.................. (3.9.1.6)}$$

Where, Sh_L is the share of labour.

Contrasted with the divisia index Solow used, Tornqvist index is another important variant of the divisia index. Under the specification of a translog production function under constant returns to scale, Diewart (1976) proved that the Tornqvist index is the exact measure of technical change. Thus, if there is a transcendental logarithmic production function as

$$\ln Y = \alpha_0 + \beta_t t + 0.5.\beta_{tt} t^2 + \sum_{i=1}^{k} \alpha_i \ln X_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln X_i \ln X_j + \sum_{i=1}^{k} \beta_{it} t \ln X_i \text{ (3.9.1.7)}$$

The Tornqvist approximation of the divisia index as introduced by Jorgensen and Grilliches (1967), can be written as

$$\overline{DI}_t = \ln \left(\frac{Y_t}{Y_{t-1}} \right) - \sum_{i=1}^{k} \overline{Sh}_i \ln \left(\frac{X_{i,t}}{X_{i,t-1}} \right) \text{ ....(3.9.1.8)}$$

Where, $\overline{Sh_i} = \frac{1}{2} \left[Sh_{i,t} + Sh_{i,t-1} \right]$. The average rate of technical change, $\overline{DI}_t$, is also called translog index of technical change.

It should be noted that the translog measure of the total factor productivity growth is not significantly different from the Solow residual measure under two conditions. First, the elasticity of substitution is not significantly different from one. Second, variation in the growth rates of inputs over time is not significant (see, Ahluwalia 1991).

Using equation (3.8.1.7) and (3.8.1.8), we shall compute the growth of total factor productivity. Total factor productivity and the rate of technical progress are synonymous.

The discrete time version of Tornqvist index is presented below.

$$\begin{aligned}\Delta \ln TFP = \Delta \ln Y(t) - [(sh_L(t) + sh_L(t-1))/2].\Delta \ln L(t) - \\ [(\{1 - sh_L(t)\} + \{1 - sh_L(t-1)\})/2].\Delta \ln K(t)\end{aligned} \quad (3.9.1.9)$$

The higher the rate of technical progress, the higher will be the growth of output. Hence, the estimation of the rate of technical progress and its input bias is relevant. Under the specification of production function as (3.9.1.7), the expression for the rate of technical progress is given as

$$\frac{\partial \ln Y}{\partial t} = \alpha_t + \beta_{tt} t + \sum \beta_{it} \ln X_i \quad (3.9.1.10)$$

Where, α_t stands for the rate of autonomous growth of total factor productivity, β_{it} for the bias in the growth of total factor productivity and β_{tt} for the rate of change in the growth of total factor productivity. If $\beta_{it} = 0$, technical progress is Hicks neutral. If $\beta_{it} > 0$, technical progress is non-neutral in the Hicksian sense and is biased with respect to the *i*-th input.

For empirical estimation, we have used ASI data for Tea processing industries of Assam for a period of 18 years (1991-2009). Nominal values were deflated by appropriate wholesale price indices from RBI: *Report on Currency and Finance* (various issues). The price indices of machinery and equipment were used to deflate fixed capital stock at current. We measure labour in terms of number of workers engaged in production.

Admittedly there is no satisfactory or universally accepted way of measuring capital stock.

Since measurement of true economic depreciation is a very complex exercise we choose to work with estimates of gross fixed capital stock. In this study, we have computed gross fixed capital stock at constant prices by using the perpetual inventory accumulation (PIA) method (Goldsmith, 1951). As regards the gross fixed capital stock at replacement cost for the benchmark year (1980-81), we have used the rule of thumb after Roychaudhury (1977), "…doubling the value of fixed capital stock at book value at current prices for the benchmark year…" to estimate the replacement cost figures of machinery and equipment.

3.9.2 Malmquist TFP Index

Malmquist TFP index, first defined in a consumer index by Malmquist (1953), and then proposed as a productivity index by Caves *et. al*.(1982). Compared to indices like the Fisher ideal index and the Tornqvist index , the Malmquist index has several advantages:

(i) No assumptions regarding market structure or economic behaviour , e.g. cost minimizing or profit maximizing , needs to be made in construction of Malmquist index;

(ii) Malmquist index requires no information on prices;

(iii) Malmquist index corresponds to very general structure of technology since it requires less restrictive assumptions than other commonly used indices;

(iv) It allows for inefficiency and thus, allows to decompose productivity growth into two mutually exclusive and exhaustive components: changes in technical efficiency over time and shifts in technology over time. This component lends themselves in a natural way to the identification of innovation, respectively; and

(v) It does not require econometric estimation, but can be implemented with a data envelopment analysis (DEA).

Caves, Christensen and Diewert (1982) defined the Malmquist productivity index (output based) as the ratio of two (output) distance functions. Distance functions are functions represents of multiple-output, multiple-input technology, which require data only on input and output quantities.

Suppose that for each time period t=1, 2... T the production technology S^t is given. The technology $S^t = \{(X^t, Q^t): X^t \ can\ produce\ Q^t\}$, describes all feasible pairs of input-output vectors. It is assumed that the constant returns to scale prevail so that output based technical efficiency equals the input-based technical efficiency index. Following Shephard (1970) and Fare (1988), the output distance function at point t is defined as:

$$D_0((X^t, Q^t) = \inf\{\theta: (X^t, Q^t)\,/\theta)\epsilon S^t\} \quad (3.9.2.1)$$

$$=[\sup\{\theta: (X^t, \theta Q^t)\,/\theta)\epsilon S^t]^{-1} \quad (3.9.2.2)$$

This function is homogeneous of degree+1 in outputs and it is the reciprocal of output based Farrell (1957) measure of technical efficiency. It completely characterize the technology in the sense that $(X^t, Q^t)\epsilon S^t$ if and only if $D_0((X^t, Q^t) \leq 1$.

To define the Malmquist index, mixed time period distance functions must be introduced. In particular,

$$D_0^{t+1}(X^t, Q^t) = \inf\{\theta: (X^t, Q^t)\,/\theta)\epsilon S^{t+1}\} \quad (3.9.2.3)$$

and

$$D_0^{t}(X^{t+1}, Q^{t+1}) = \inf\{\theta: (X^{t+1}, Q^{t+})\,/\theta)\epsilon S^{t}\} \quad (3.9.2.4)$$

The terminology 'mixed period' is used since information from t and t+1 is required to define equations (3.9.2.3) and (3.9.2.4). Again, $D_0^{t+1}((X^t, Q^t) \leq 1$ if and only if $(X^t, Q^t)\epsilon S^{t+1}$, and similarly for $D_0^{t}(X^{t+1}, Q^{t+1})$.

Caves, Christensen and Diewert (1982) defined the output based Malmquist productivity index as:

$$\boldsymbol{M^t} = \frac{D_0^t(X^{t+1}, Q^{t+1})}{D_0^{t+1}(X^t, Q^t)} \qquad (3.9.2.5)$$

3.10 Data

The present study uses secondary data primarily from two sources. (i) Annual Survey of Industries: *Summary Results for Factory Sector* (various issues) in order to get industry level long run time series data on Tea Manufacturing for the period 1991 – 2009 (NIC – 2001 5 Digit Classification Code: 22710), and (ii) Annual Balance Sheet information as per CSO (Central Statistical Organisation) format at the Estate level, submitted to the Tea Board of India on an annual basis. The study period is 2001- 2010. The study targets 31 firms, around 17 from upper Assam and the remaining from the three districts of Barak Valley. The selection of the gardens is on the basis of convenient sampling. The sample for the present study is entirely non-random. The data in its present format is too difficult to obtain at the garden level. During the course of the survey as many as 85 gardens of both Barak valley and that of Upper Assam were visited for relevant information about inputs and output. Most of the managerial staff declined to provide such detailed data at the garden level. In fact just 31 gardens responded positively by furnishing the desired data on relevant inputs and output over the study period. Thus there may be a certain degree of non-randomness in the data which under the present circumstances was unavoidable. It is possible that either the board of directors and/or the garden owners strictly instruct factory managers not to divulge balance sheet related data to any third party and that includes academic institutions and researchers as well. From this reaction of garden owners and managers it is apparent that owners and the members of the board of directors are inclined more towards suppression of information related real output, revenue, costs, profits and profits after taxes. Thus garden level and factory level data are difficult to obtain.

Chapter Four

EMPIRICAL RESULTS AND DISCUSSION