

DECLARATION

I, **Amrita Bhattacharjee**, bearing Registration Number.-Ph.D/2054/12 dated 12/09/2012, hereby declare that the subject matter of the thesis entitled "**Computational Characteristics of Words over Formal Languages**" is the record of works done by me and that the contents of the thesis did not form the basis for award of any other degree to me or to anybody else to the best of my knowledge. The thesis has not been submitted in any other University / Institute. This thesis is being submitted to Assam University for the degree of Doctor of Philosophy in Computer Science.

(AMRITA BHATTACHARJEE)

Research Scholar

(Ph.D. Registration No.-Ph.D/2054/12)

Place:

Date:

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— Amrita Bhattacharjee

Dedication

**This thesis is dedicated to my beloved
parents**

***Smti Archana Bhattacharjee and
Shri Lakshmikanta Bhattacharjya,***

parents-in-law

***Smti Nilima Chaudhuri and Late Nikhiles Chaudhuri,
husband***

Biplab Chaudhuri,

son and daughter

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*For their endless love, support, encouragement and sacrifice ,
without whom none of my success would be possible.*

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List of Symbols

| | |
|----------------------|---|
| \emptyset | The empty set |
| \mathbb{N} | The set of natural numbers |
| \mathbb{Z} | The set of integers |
| FSA | Finite state automata |
| NFSA | Non-deterministic finite state automata |
| DFSA | Deterministic finite state automata |
| Σ | An ordered alphabet |
| Σ^* | The set of words formed from Σ |
| $ w _{a_i}$ | The number of occurrences of a_i in a word $w \in \Sigma^*$ |
| $\Psi_{M_n}(\zeta)$ | $n \times n$ Parikh matrix over an word ζ |
| m_{ij} | The element in the i^{th} row and j^{th} column of a matrix |
| \sim_r | Ratio property |
| \sim_{wr} | Weak ratio property |
| NLP | Natural language processing |
| $R(\zeta)$ | M-ambiguity reduction factor of the word ζ |
| $d_S(\alpha, \beta)$ | Stepping distance between the words α and β |
| CFG | Context-free grammar |
| i.e. | That is |

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