

## Appendix 2: Proof of Transaction Cost Models

Here, the study proof that all three models turn into the Perold [1988] model and have the same result.

IS, according to Perold [1988], is given in Equation (7) in the text. The study shows here that the models from Wagner and Edwards [1993] and Kissell [2006], as well as our proposed expanded model, also offer the same result as IS proposed by Perold [1988].

IS, according to Wagner [1991] and Wagner and Edwards [1993], is given as follows:

$$IS = \text{Delay-related cost} + \text{Trading-related cost} + \text{Opportunity cost} + \text{Fixed} \quad (\text{A3.01})$$

$$\left\{ \begin{array}{l} \text{DC or TC} \\ = \text{TRDC} + \text{ORDC} \end{array} \right\} + \left\{ \begin{array}{l} \text{TC or PI} \\ = \text{SC} + \text{MIC} + \text{PAC} + \text{TRC} \end{array} \right\} + \text{OC} + \text{Fixed} \quad (\text{A3.02})$$

Mathematically, it can be presented as follows:

$$IS = \left\{ \begin{array}{l} \sum x_j (P_0 - P_d) \\ + (X - \sum x_j) (P_0 - P_d) \end{array} \right\} + \left[ \begin{array}{l} \left[ \begin{array}{l} (P_{bj} - P_{mj}) \\ + (P - P) \end{array} \right] \\ + (P^j - P^j) \end{array} \right] + (X - \sum x_j) (P_n - P_0) + F \quad (\text{A3.03})$$

where  $\sum x_j (P_0 - P_d) = \text{Trading-related delay cost}$  and  $(X - \sum x_j) (P_0 - P_d) = \text{Operation-related delay cost}$ . Together, this makes the total delay-related cost the first component.

Trading-related cost has four components that are defined as  $\sum x_j (P_{bj} - P_{mj})$   
= Spread cost;  $\sum x_j (P_j - P_{bj})$  = Market impact cost;  $\sum x_j (P_{pj} - P_0)$  = Price  
appreciation cost; and  $\sum x_j (P_{mj} - P_{pj})$  = Timing risk cost. The third component is  
the opportunity cost, defined as  $(X - \sum x_j)(P_n - P_0)$ ; and finally,  $F$  = Fixed  
charges.

Equation (A3.03) can be reduced to the following form:

$$\begin{aligned}
IS &= X(P_0 - P_d) + \sum x_j (P_j - P_0) + (X - \sum x_j)(P_n - P_0) + F \\
&= XP_0 - XP_d + \sum x_j P_j - \sum x_j P_0 + XP_n - XP_0 - \sum x_j P_n + \sum x_j P_0 + F \\
&= \sum x_j P_j - \sum x_j P_n + XP_n - XP_d + F \\
&= XP_n - XP_d - \sum x_j P_n + \sum x_j P_j + F \\
&= (XP_n - XP_d) - (\sum x_j P_n - \sum x_j P_j - F) \\
&= \text{Perold [1988]}
\end{aligned}$$

IS, according to Kissell [2006], is given as follows:

$$IS = \text{Investment related} + \text{Trading related} + \text{Opportunity cost} + \text{Fixed} \quad (\text{A3.04})$$

$$IS = IRC + \left\{ \text{TC} = \text{SC} + \text{MIC} + \text{PAC} + \text{TRC} \right\} + \left\{ \text{OC} = \text{IROC} + \text{TROC} \right\} + \left. \begin{array}{c} \text{Fixed} \\ \text{Charges} \\ \text{C} + \text{F} + \text{R} + \text{T} \end{array} \right\} \quad (\text{A3.05})$$

Mathematically, it can be written as follows:

$$IS = \sum_j x_j (P_0 - P_d) + \left[ \sum_j x_j \left\{ \begin{matrix} (P_{bj} - P_{mj}) \\ + (P_j - P_0) \end{matrix} \right\} \right] + (X - \sum_j x_j) \begin{pmatrix} (P_n - P_0) \\ + (P_0 - P_d) \end{pmatrix} + F \quad (A3.06)$$

Here, investment-related cost is  $\sum_j x_j (P_0 - P_d)$  and the trading-related subcomponents are identical to those given by Wagner and Edwards [1993], shown in Equation (A3). Unlike Wagner and Edwards [1993], the study describe opportunity cost as having two subcomponents that are defined as  $(X - \sum_j x_j)(P_n - P_0) =$  Trading-related opportunity cost and  $(X - \sum_j x_j)(P_0 - P_d) =$  Investment-related opportunity cost. The last component is fixed cost, which has more components than are given Wagner and Edwards [1993].

Again, Equation (A3.06) can be reduced to the following form:

$$IS = \sum_j x_j (P_0 - P_d) + \sum_j x_j (P_j - P_0) + (X - \sum_j x_j)(P_n - P_d) + F \quad (A3.07)$$

This can be rearranged as follows:

$$\begin{aligned} IS &= \sum_j x_j P_j - \sum_j x_j P_n + XP_n - XP_d + F \\ &= XP_n - XP_d - \sum_j x_j P_n + \sum_j x_j P_j + F \\ &= (XP_n - XP_d) - \left( \sum_j x_j P_n - \sum_j x_j P_j - F \right) \\ &= \text{Perold [1988]}. \end{aligned} \quad (A3.08)$$

Finally, IS, according to the proposition of this study, is given as follows:

$$IS = \text{Investment-related cost} + \text{Trading-related cost} + \text{Opportunity cost} + \text{Fixed} \quad (\text{A3.09})$$

$$IRC + \{TC = SC + MIC + PAC + TRC\} + \{OC = IROC + FTROC + RTROC\} + \left\{ \begin{array}{l} \text{Fixed} \\ = C + F + R + T \end{array} \right\} \quad (\text{A3.10})$$

Mathematically, it can be written as follows:

$$IS = \sum_j x_j (P_0 - P_d) + \left\{ \begin{array}{l} \left[ \begin{array}{l} (P_{bj} - P_{mj}) \\ + (P_0 - P_d) \end{array} \right] \\ \left[ \begin{array}{l} (P_{pj} - P_0) \\ + (P_0 - P_d) \end{array} \right] \\ \left[ \begin{array}{l} (P_0 - P_d) \end{array} \right] \\ \left[ \begin{array}{l} (P_{mj} - P_{pj}) \end{array} \right] \end{array} \right\} + \left\{ \begin{array}{l} (X - \sum_j x_j)(P_0 - P_d) \\ + (X - \sum_j x_j)(P_1 - P_0) \\ + (X - \sum_j x_j)(P_n - P_1) \end{array} \right\} + F \quad (\text{A3.11})$$

Here, investment-related cost is  $\sum_j x_j (P_0 - P_d)$ , as in Kissell [2006], and trading-related subcomponents are identical to those given by Wagner and Edwards [1993], shown in Equation (A3.03). The opportunity cost has three subcomponents and are defined as  $(X - \sum_j x_j)(P_0 - P_d) =$  Investment- or operation-related opportunity cost,  $(X - \sum_j x_j)(P_1 - P_0) =$  First trading-related opportunity cost, and  $(X - \sum_j x_j)(P_n - P_1) =$  Residual trading-related opportunity cost.

Equation (A9) can be reduced to the following:

$$\begin{aligned} &= \sum_j x_j (P_0 - P_d) + \sum_j x_j (P_j - P_0) + (X - \sum_j x_j)(P_n - P_d) + F \\ &= XP_n - XP_d - \sum_j x_j P_n + \sum_j x_j P_j + F \\ &= (XP_n - XP_d) - \left( \sum_j x_j P_n - \sum_j x_j P_j - F \right) \\ &= \text{Perold [1988]}. \end{aligned} \quad (\text{A3.12})$$

### Appendix 3: Exhibits of Chapter 3

Exhibit 3.01: Classification of Transaction Costs/IS

Perold [1988]		Wagner and Edwards [1993]		Kissell [2006]		Classifications of the Study	
Cost Name		Cost Name		Cost Name		Cost Name	
IS = PR – RR		<b>Timing cost</b>		<b>Investment-related cost</b>		<b>Investment-related cost</b>	
		TRDC		IRC		IRC	
		ORDC					
		<b>Price impact</b>		<b>Trading-related cost</b>		<b>Trading-related cost</b>	
		SC		SC		SC	
		MIC		MIC		MIC	
		PAC		PAC		PAC	
		TRC		TRC		TRC	
		<b>Opportunity cost</b>		<b>Opportunity cost</b>		<b>Opportunity cost</b>	
		OC		IROC		IROOROC	
				TROC		FTROC	
						RTROC	
<b>Fixed</b>	<i>Fixed</i>	<b>Fixed</b>	<i>Fixed</i>	<b>Fixed</b>	<i>Fixed</i>	<b>Fixed</b>	<i>Fixed</i>
		C		C	<i>Fixed</i>	C	<i>Fixed</i>
				F	<i>Fixed</i>	F	<i>Fixed</i>
				R	<i>Fixed</i>	R	<i>Fixed</i>
				T	<i>Fixed</i>	T	<i>Fixed</i>

IS = implementation shortfall; PR = paper return; RR = real portfolio return; TRDC = trading-related delay costs; ORDC = operation-related delay costs; SC = spreads costs; PAC = price appreciation costs; MIC = market impact costs; TRC = timing risk costs; OC = opportunity costs; IROC = investment-related opportunity costs; IROOROC = investment-related or operation-related opportunity costs; TROC = trading-related opportunity costs; FTROC = first trading-related opportunity costs; RTROC = residual trading-related opportunity costs; C = commission; F = fees; R = rebates; T = taxes.

*Exhibit 3.02: National Best Bid and Offer Prices*

	$t_1$	$t_2$	$t_3$
Best offer $P_b$	\$101.50	\$102.70	\$103.80
Best bid $P_w$	\$101.20	\$102.10	\$103.60
Midpoint price $P_m$	\$101.35	\$102.40	\$103.70

*Exhibit 3.03: Timeline of Events*

09:30 a.m.		10:30 a.m.		11:00 a.m.		11:30 a.m.		02:30 p.m.		04:00 p.m.
Market starts		Investment decision time		Order release time		Time at which the broker begins to implement the order		Time at which the broker stops trading		Market close
$(P_s)$		$(P_d)$		$(P_0)$		$(P_1)$		$(P_k)$		$(P_n)$

Exhibit 3.04: Calculations for the Four Models

Perold [1988]		Wagner and Edwards [1993]		Kissell [2006]		proposed classification of the Study	
Cost name	Costs in bps	Cost name	Costs in bps	Cost name	Costs in bps	Cost name	Costs in bps
IS = PR – RR	315	Timing cost TRDC	90	Investment-related cost IRC	90	Investment-related cost IRC	90
		ORDC	10				
		Price impact SC	18	Trading-related cost SC	18	Trading-related cost SC	18
		MIC	28	MIC	28	MIC	28
		PAC	130.50	PAC	130.50	PAC	130.50
		TRC	13.50	TRC	13.50	TRC	13.50
		Opportunity cost OC	25	Opportunity cost IROC TROC	10 25	Opportunity cost IROOROC FTROC RTROC	10 10 15
<b>Fixed</b>	<i>Fixed</i>	<b>Fixed</b> C	<i>Fixed</i>	<b>Fixed</b> C F R T	<i>Fixed</i> <i>Fixed</i> <i>Fixed</i> <i>Fixed</i>	<b>Fixed</b> C F R T	<i>Fixed</i> <i>Fixed</i> <i>Fixed</i> <i>Fixed</i>
<b>Total IS =</b>	<b>315 + Fixed</b>	<b>Total IS =</b>	<b>315 + Fixed</b>	<b>Total IS =</b>	<b>315 + Fixed</b>	<b>Total IS =</b>	<b>315 + Fixed</b>