PG (CBCS) ODD SEMESTER EXAMINATION, 2022

PHYSICS

3rd Semester

Course No. : PHYCC - 301

(Mathematical Physics-II)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer any five questions, taking one from each unit)

UNIT - I

1. (a) Find the general solution in the form of a function F(x, y) = c for the equation

$$\frac{dy}{dx} = -\frac{4x^3 + 6xy + y^2}{3x^2 + 2xy + 2}$$ 4

(b) Solve the equation $y^3\frac{dy}{dx} + \frac{y^4}{x} - y^3 = 0$ 4

(c) Find two linearly independent solution of the system of first order linear differential equation

$\frac{dx}{dt} = 5x + 4y$ *and* $\frac{dy}{dt} = -x + y$ 6

2. (a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = cosx$ 4

(Turn Over)

(b) Solve $e^{Y}\left(\frac{dy}{dx}+1\right)=1$ with initial condition $y(0)=1$ using integrating factor. 4

(c) Find the power series solution of the differential equation $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2+2)y=0$ 6

UNIT - II

3. (a) Determine the fixed points of the cubic map $x_{n+1}=ax_n^3+(1-a)x_n$ for $0<a<4$ & $0<x_n<1$ 2

(b) For each fixed points, determine the range of values of 'a' for which fixed point is stable 4

(c) For the legistic map $x_{n+1}=\mu x_n(1-x_n)$, $x_n \in [0, 1]$ and $1<\mu<4$, show that the second bifurcation that leads to cycles of period 4 is located at $\mu = 1+\sqrt{6}$. 3

(d) Define Lyapunov exponent. Show the relation between onset of Chaos & value of Lyapunov exponent. 5

4. (a) Explain transcritical bifurcation and pitchfork bifurcation. 4

(b) Determine the stability of fixed points to the logistic growth equation $\frac{dN}{dt}=rN\left(1-\frac{N}{K}\right)$ for r>0, where symbols have their usual meaning. 4

(c) Write Lorenz equation and explain how chaos originates from it. 6

(c) Explain binomial distribution. Prove that the most probable no. of success in n independent trials in binomial group is [Np + P], where [.] denotes the integral part. 2+4

10. (a) Define mean value, variance for both discrete random variable and continuous random valuable. 2

(b) Prove Chebychev inequality

$$P(|x-<x>|\,2\,k\sigma)<\frac{1}{k^2}$$

where notations have usual meaning 6

(c) Define normal distribution. 2

(d) If two normal distributions have the same total frequency but the standard deviation of one is k times that of the other, show that the maximum frequency of the first is $\frac{1}{k}$ times that of the other. 4

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(b) Prove that a group whose order is a prime number must be a cyclic group. 2

(c) Explain subgroup with example. 2

(d) State and provel Lagrange's theorem. 1+3

(e) Define class of a group. Prove that no element can be common between two distinct classes. 1+3

8. (a) For the group (known as dihedral group D_3) which is the group of symmetry transformation on an equilateral triangle, construct the irreducible representation. 7

(b) Show that the number of inequivalent irreducible of a group is equal to the number of classes in the group. If the dimensions of these representations are { n_1 , n_2 , ... } then $\sum_i n_i^2 = g$, g being the order of the group. 7

UNIT - V

9. (a) Prove P (A∪B∪C) = P (A) + P (B) + P (C) - P (A∩B) - P (A∩C) - P (B∩C) + P (A∩B∩C)

when notations have usual meaning. 4

(b) State and prove Baye's theorem. 4

$$P(A|B) = \frac{P(A)}{P(B)} = P\,(B/A)$$

UNIT - III

5. (a) Solve the differential equation given by $\frac{d}{dx}\left(p(x)\frac{dy}{dx}\right) + q\,(x)y = f(x)$ using Green's function, where the boundary values are given by $y = 0$ at x = a, y = 0 at x = b 9

(b) Find the electrostatic potential $\phi\,(\bar{r})$ which satisfies Poisson's equation $\nabla^2\phi\,(\bar{r}) = -4\pi\,p(\bar{r})$ 5

6. (a) The Legendre polynomials $P_l(x)$ is defined by the following relation $(1 - 2xt + t^2)^{-1} = \sum_{l=0}^{\infty} t^l P_l\,(x)$ t<1.

Show that

i) $xP_l'\,(x) - lp_l(x) = P_{l-1}'(x)$

ii) $xP_{l-1}'\,(x) + lp_{l-1}(x) = P_l'(x)$

iii) $(x^2 - 1)\,P_l'\,(x) = xlp_l(x) - lP_{l-1}(x)$ 6

(b) Show that $\int_{-1}^{1} P_n\,(x)p_n\,(x)\,dx = \frac{2}{2n+1}\,\delta_{mn}$ 4

(c) Prove Rodrigues' formula $P_l\,(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}\,(x^2 - 1)^l$ 4

UNIT - IV

7. (a) Define cyclic group with example. 2