Prove the following commutators :

(a) $[\hat{L}\pm, \hat{R}\pm] = \pm 2\hbar\hat{z}$ *and*

(b) $[\hat{L}\pm, \hat{R}_{\mp}] = 0$

(c) What do you mean Clebsch-Gordan coefficients? 4

10. Conisder the case where $j = 1$ 14

(a) Find the matrices representing the operators $\vec{J}^2, \hat{J}_z, \hat{J}_\pm, \hat{J}_x$ *and* $\hat{J}_y$.

(b) Find the joint eigenstates of $\vec{J}^2$ *and* $\vec{J}_z$ and verify that they form an orthonormal and complete basis.

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PG (CBCS) ODD SEMESTER EXAMINATION, 2022

PHYSICS

1st Semester

Course No. : PHYCC - 103

(Quantum Mechanics-I)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer any five questions, taking one from each unit)

UNIT - I

1. (a) State and prove Ehrenfest's theorem. 10

(b) State the postulates of quantum mechanics. 4

2. (a) Show that the maximum kinetic energy transferred to a proton when hit by a photon of frequency ν is given by

$$Kinetic\ energy = \frac{h\nu}{[1 + m_p c^2/2h\nu]}$$

where m_p, c & h are mass of proton, speed of light and planck's constant respectively. 6

(Turn Over)

(b) Discuss the Eintein's explanation of photoelectric effect. 6

(c) What is de Broglie's hypothesis. 2

UNIT - II

3. Obtain Schroedinger's (i) time-dependent (ii) time-independent equations for matter waves. Discuss the physical significance of wave function. 14

4. A particle is moving in a one-dimensional potential given by 14

$V = 0$ for $x < 0$

$V = V_0$ for $x \geq 0$

(a) Write down the Schroedinger wave equation for the particle and solve it.

(b) Calculate the transmittance & reflectance for the case (i) $E > V_0$ and (ii) $0 < E < V_0$, where E is the total energy of the particle.

UNIT - III

5. (a) State and prove generalized ucertainty principle. 10

(b) Derive Heisenberg's uncertainty relation by using generalized uncertainty principle. 4

6. Explain Heisenberg, Schroedinger and interaction pictures in quantum mechanics. 14

UNIT - IV

7. (a) Explain infinitesimal and finite unitary transformations. 7

(b) Let U be a transformation matrix which connects two complete and orthonormal bases $\{|\phi_n>\}$ and $\{|\phi_n'>\}$. Show that U is unitary. 7

8. (a) Establish a connection between the position and momentum representation. 7

(b) State and prove Parseval's theorem. 3

(c) Describe the matrix representation of an operator. 4

UNIT - V

9. (a) Obtain the commutation relations satisfied by the orbital angular momentum operators $\hat{L}_x, \hat{L}_y$ *and* $\hat{L}_z$. 4

(b) If $\hat{L}\pm$ *and* $\hat{R}\pm$ are defined by 6

$\hat{L}\pm = \hat{L}_x \pm i\,\hat{L}_y$ *and*

$\hat{R}\pm = \hat{x} \pm i\hat{y}$

(Turn Over)