(iii) The vector

$U = (2x^2 + 8xy^2z)\,\hat{\imath} + \,(3x^3y - 3xy)\,\hat{\jmath} - \,(4y^2z^2 + 2x^3z)\,\hat{k}$

is not a solenoidal one. 3

(iv) $\vec{\nabla} \times [(\hat{e} \times \hat{r}) \times \hat{e}] = 0$, $\hat{e}$ being a unit vector 3

(v) Show that $r^n\,.\,\vec{r}$ is an irrotational vecor for any value of n, but is solenoidal ony if n = -3. 3

10. (a) What is orthogonal curvilinear coordinate system? 2

(b) For spherical polar co-ordinates, verify the mutual orthogonality of $\frac{\partial \vec{r}}{\partial r}$ $\frac{\partial \vec{r}}{\partial \theta}$ *and* $\frac{\partial \vec{r}}{\partial \phi}$ 4

(c) Compute $\oint [(xy - x^2)\,dx + x^2y\,dy\,]$ over the triangle bounded by lines $y = 0$, $x = 1$ and $y = x$ and hence verify Green's theorem. 5

(d) What is a tensor? What is meant by rank of a tensor? Define contravariant and co-variant tensors. 3

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PG (CBCS) ODD SEMESTER EXAMINATION, 2022

PHYSICS

1st Semester

Course No. : PHYCC - 102

(Mathematical Physics)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer any five questions, taking one from each unit)

UNIT - I

1. (a) Define linear vector space. What is meant by the field of the linear vector space ? 2+1=3

(b) Show that an arbitrary non-trivial vector V in V^n space can be expressed uniquely as a linear combination of n linearly independent vectors in V^n. 5

(c) What is orthonormal basis? Show that in such a basis the inner product of two vectors is completely determined by the components of these two vectors in that basis. 1+3=4

(Turn Over)

(d) If $|e_i>$ is a given orthonormal basis ket vector, how can you find the components of any vector $|v>$ in that basis? 2

2. (a) Write the commutation relation between two operators Ω and Θ. When these two operators are said to commute? 2

(b) Define the following terms 2+2=4

(i) Inverse of an operator

(ii) Adjoint of an operator

(c) What is a unitary operator? Show that unitary operator preseve the inner product between vectors they act on. 1+3=4

(d) Write down Schwartz inequality relation for two areal integrable functions. Prove the in equality. 1+3=4

UNIT - II

3. (a) What are single and multiple valued of function? Give an example of multiple valued function explaining different branche of it. 1+3=4

(b) Define analytic function. When a function can be analytic in a region? Prove the necessity of Cauchy-Riemann equations. 1+2+3=6

UNIT - IV

7. (a) Discuss the effect of changing origin and scale on Simpson's $^1/_3$ and $^3/_8$ rule. 7

(b) Estimate the value of

$$\int_0^{\pi/2} Sin\, x\, dx \quad \text{by}$$

(i) Trapezoidal rule

(ii) Simpson's rule using 11 ordinates. 3+4=7

8. (a) What is meant by roots of a function? How do you estimate the roots of a given function? Explain the Bisection and Newton Raphson methods for estimating roots of a function. Using Newton Raphson method find the root of $f(x) = \cos x - x$ 1+1+2+3+3=10

(b) Discuss the Range Kutta Method for solution of first order differential equations. 4

UNIT - V

9. Prove that

(i) $\nabla^2 (uv) = u\, \nabla^2 v + 2\, \nabla u \,.\, \nabla v + v\, \nabla^2 u$ 2

(ii) $\nabla^2 (^1/_r) = 0$ where, $r^2 = x^2 + y^2 + z^2$ 3

(i) $f(t) = \sin {}^{\pi t}/_{T}$ 2+4

(ii) $f(t) = \left({}^{t^2}/_{T^2} \right)$

(b) Find the Fourier transform of 4

(i) $\dfrac{1}{x^2 + a^2}$

(ii) e^{-r^2/a^2}

where a is a constant and $r^2 = x^2 + y^2 + z^2$

(c) Find the Laplace transform of 4

(i) $F(t) = e^{at}$ Sin bt

(ii) $F(t) = e^{at}$ Cos bt

6. (a) Define integral transform of a function. 2

(b) Find the Laplace transform of 3x2=6

(i) t^{n}

(ii) e^{at}

(iii) sin ht

(c) State the the condition under which a function can be expanded in the form of a Fourier series. Express the following function in a Fourier series. 2+4=6

$$F(x) = x + x^2 \; ; \; -\pi < x < \pi$$

(c) Given $(x \text{ Sin } y - y \text{ Cos } y)\, e^{-x} = u$

(i) Prove that u is a harmonic function

(ii) Find v such that the function $f(z) = u + iv$ is analytic. 2+2=4

4. (a) Prove the Cauchy's integral theorem

$$\oint_e f(z)\, dz = 0$$

if $f(z)$ is analytic with derivative $f'(z)$ which is continuous at all points inside the curve C. 3

(b) Expand $f(z) = Sin\ z$ in a Taylor series about $z = \pi/4$ 3

(c) Find the Laurent series about the indicated singularity for a given functions 2+2=4

(i) $\dfrac{e^{2z}}{(z-1)^3}\ ; \ z = 1$

(ii) $(z-3)\sin\dfrac{1}{z+2}\ ; \ z = -2$

(d) Evaluate 4

$$\frac{1}{2\pi l} \oint \frac{e^{zt}}{z^2\,(z^2 + 2z + 2)}\, dz$$

around the circle with $|z|=3$

UNIT - III

5. (a) Discuss the role of Fourier series in Physics. Obtain fourier series of expansions of