motion of a body falling vertically in a uniform gravitational field. 5+9 = 14

UNIT - V

9. (a) Considering the rotation of a rigid body about a point in it derive the expression for inertia tensor. If the rigid body is not composed of discrete particles i.e., in case of continuous body what would be the inertia elements?

(b) Derive Euler's equations of motion of a rigid body.

8+6=14

10. What do you mean by stable and unstable equilibria? A double pendulum consists of a pendulum of mass 'm_1' and length 'l_1' to which a second pendulum of mass 'm_2' and length 'l_2' is suspended. Find the expression for normal mode of frequencies.

2+12=14

★★★

PG ODD SEMESTER EXAMINATION, 2022

PHYSICS

1st Semester

Course No. : PHYCC - 502

(Classical Mechanics)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer any five questions, taking one from each unit)

UNIT - I

1. (a) What is the center of mass of a system? Prove that the magnitude R of the position vector for the center of mass from an arbitrary origin is given by the equation:

$$M^2 R^2 = M\sum_i m_i\, r_i^2 - \frac{1}{2}\sum_{i,j} m_i\, m_j\, r_{ij}^2$$

[Where symbols have their usual meanings]

(b) Prove that the center of mass of two particles lies on the line joining the particles and between them.

(c) Show that the kinetic energy of a system of particles can be expressed as the sum of the kinetic energy of the motion and the kinetic energy about the motion. 4+5+5=14

(Turn Over)

2. (a) Find the conditions for stability of orbit under central force motion.

   (b) A particle describes a circular orbit under the influence of an attractive central force directed towards a point on the circle. Show that the force varies as inverse fifth power of the distance. 7+7=14

UNIT - II

3. (a) What type of difficulties arise due to the constraints in the solution of mechanical problems and how these are removed?

   (b) Show that the constraints of a rigid body are conservative in nature.

   (c) Two points of mass 'm' are joined by a rigid weightless rod of length '*l*', the center of which is constrained to move on a circle of radius 'a'. Set up the kinetic energy of the system in generalized co-ordinates. 4+5+5=14

4. (a) State the principle of virtual work and hence derive the equation of motion of a system of particles in generalized coordinates.

   (b) Consider a mass 'm' moving in two dimensions with potential energy $U(x, y) = \frac{1}{2} k r^2$, where $r^2 = x^2 + y^2$. Write down the Lagrangian using coordinates x and y and find the equation of motion. 8+6=14

UNIT - III

5. (a) Prove that if time 't' doesn't appear in the Lagrangian L explicitly, then the Hamiltonian H of system is constant of motion, and it represents nothing, but total energy provided the system is conservative.

   (b) Using Hamilton's equations of motion show that the angular momentum is conserved for a particle moving in a central force field. 7+7=14

6. (a) If $f(x, y)$ is an arbitrary function with basis (x, y) then establish the Legendre transformation equations to change the basisof the function from (x, y) to (u, v) .

   (b) Show that the transformation $Q = \frac{1}{p}$ and $P = qp^2$ are canonical.

   (c) Show that a function whose Poisson bracket with Hamiltonian vanishes is a constant of motion. 6+4+4=14

UNIT - IV

7. Deduce Hamilton – Jacobi equation and discuss the physical significance of 'Hamilton's Principal Function'. Solve the equation of motion of a simple harmonic oscillator using Hamilton – Jacobi method. 7+7=14

8. Discuss the physical significance of 'Hamilton's Characteristic Function'. Apply Hamilton – Jacobi method to determine the