10. (a) Show that

(i) every subgroup of index 2 is normal. 3

(ii) if the elements of a group are their own inverse, then the group is abetian. 4

(ii) the group of all positive numbers under multiplication is isomorphic to the group of all real numbers under addition. 3

(b) How to construct character table of a group? 4

★★★

PG ODD SEMESTER EXAMINATION, 2022

PHYSICS

1st Semester

Course No. : PHYCC - 501

(Mathematical Physics)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer any five questions, taking one from each unit)

UNIT - I

1. (a) Define linear vector space. What is meant by the field of the linear vector space ? 2+1=3

(b) Show that an arbitrary nontrivial vector V in V^n space can be expressed uniquely as a linear combination of *n* linearly independent vectors in V^n. 5

(c) What is orthonormal basis? Show that in such a basis the inner product of two vectors is completely determined by the components of these two vectors in that basis. 1+3=4

(Turn Over)

(d) If $|e_i>$ is a given orthonormal basis ket vector, how can you find the components of any vector $|v>$ in that basis? 2

2. (a) Define conravariant and covariant tensors of 2nd rank. 2+2=4

(b) Prove the following relations 2+2=4

(i) $\frac{\partial x^p}{\partial x^q} = \delta_q^p$

(ii) $\frac{\partial x^p}{\partial x^q} - \frac{\partial \bar{x}^q}{\partial x^r} = \delta_r^p$

(c) What are Christoffel's symbols of first and second kind? Write down the laws of transformation of the Christoffel's symbols. 2+4=6

UNIT - II

3. (a) Classify first order partial differential equations into linear, semi-linear and non-linear with examples. 6

(b) Solve the following partial differential equations 2+2=4

(i) $\frac{\partial^2 z}{\partial y^2} = Sin\ (x\ y)$

(ii) $\frac{\partial^2 z}{\partial y^2} = x^2\ Sin\ (x\ y)$

(c) Given $Sin\ 45^0 = 0.7071$, $Sin\ 50^0 = 0.7660$, $Sin\ 55^0 = 0.8192$, $Sin\ 60^0 = 0.8660$ 4

Find $Sin\ 52^0$

8. (a) If u_x is a function where fifth differences are constant, $\int_{-1}^{1} u_x\, dx$ can be expressed in the form $pu_{-\alpha} + qu_0 + pu_\alpha$, find the values of p, q and α. 4

(b) Evaluate $\int_0^1 \frac{x^2}{1+x^3}$ and hence find the value log_e^2 4

(c) Apply Runge Kutta method to find y upto 4th decimal places. When $x = 0.3$ from $\frac{dy}{dx} = x + y^2$ where $y = 0$ when $x = 0$ 6

UNIT - V

9. Show that

(a) Each element in an abelian group forms a class by itself. 3

(b) Kernal of homomorphism from a group G′ to another group G is a normal subgroup of G′. 5

(c) Any representation of a finite group where marices may be non unitary is equivalent to a representation by unitary matrices. 6

(Turn Over)

(ii) Find v such that the function $f(z) = u + iv$ is analytic.

6. (a) Define Fourier transform. Write the inversion formula for Fourier transform. $1^1/_2$x2=3

(b) If $f(s)$ is a Fourier transform of $F(x)$, then prove that 2+2=4

(i) $F\{F(-x)\} = f(-s)$

(ii) $F\{\overline{F(-x)}\} = \overline{f}(s)$

where bar over a quantity represents its complex conjugate.

(c) If $F(t)$ is a function piecewise continuous on every finite interval in the range $t \geq 0$ and satisfies $|F(t)| \leq me^{\alpha t}$ for all $t \geq 0$ and for constants α and m, then prove that the Laplace transform of $F(z)$ exists. 3

(d) Find the Laplace transform of the function $F(t) = t^n$, where n is a positive integer. 4

UNIT - IV

7. (a) Obtain Poisson Distribution as a limiting care of Binomeal distribution 6

(b) Evaluate 2+2=4

(i) $\Delta^n \{ e^{(a+bx)} \}$

(ii) $\Delta^n \left(\frac{1}{x}\right)$

(c) Show that 4

$$\int_{-1}^{1} [Pn(x)]^2 \, dx = \frac{2}{2n+1}$$

where $P_n(x)$ is Legendre polynomeal of first kind.

4. (a) Form the partial differential equation by eliminating h and k from the equation 4

$(x - h)^2 + (y - k)^2 + z^2 = \lambda^2$

(b) Show that for Bessel function $Jn(x)$ satisfy

$$\frac{\partial}{\partial x}\left(\frac{J-n}{J_{-n}}\right) = \frac{-2 \, Sin \, n \, \pi}{\pi \, x \, J_{n^2}}$$ 4

(c) For Legendre polynomeal of first kind, show that

$$\int_{-1}^{1} (x^2-1)^2 \, P_{n+1}(x) \, P_n'(x) \, dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$$ 6

UNIT - III

5. (a) What are single and multiple valued functions? Give an example of multiple valued function explaining different branches of it. 1+3=4

(b) Define analytic function, when a function can be analytic in a region? Prove the necessity of Cauchy-Reiemann equations. 1+2+3=6

(c) Give $u = e^{-x}(x \, Sin \, y - y \, Cos \, y)$ 2+2=4

(i) Show that u is a harmonic function

(Turn Over)