UG Even Semester (CBCS) Exam., May—2017

MATHEMATICS

(Pass)

(4th Semester)

Course No. : BSMP–402

(Differential Calculus, Integral Calculus and Differential Equation)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) If $ax^2 + 2hxy + by^2 = 1$, then prove that

$$\frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx + by)^3}$$ 3

(b) If $y = e^{a \sin^{-1} x}$, then prove that

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - (n^2 + a^2) y_n = 0$$

Hence find the value of y_n, when $x = 0$. 6

(c) Prove that (if $0 < a < b < 1$)

$$\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$$

Hence show that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$$ 5

OR

2. (a) If $y = x \log \frac{x-1}{x+1}$, then show that

$$y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$$ 3

(b) If $V_n = \frac{d^n}{dx^n}(x^n \log x)$, then show that

$$V_n = nV_{n-1} + (n-1)!$$

Hence show that

$$V_n = n!\left[\log x + 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right]$$ 6

(c) State and prove Lagrange's mean value theorem. Give geometrical interpretation of Lagrange's mean value theorem. 5

3. (a) Using Maclaurine's series, expand $\tan x$ up to the term containing x^5. 5

(b) Evaluate : 3

$$\underset{x\to 0}{\text{Lt}} \frac{x\cos x - \sin x}{x^2 \sin x}$$

(c) Find the equation of the tangent at any point (x_1, y_1) to the curve $x^{2/3} + y^{2/3} = a^{2/3}$. Show that the portion of the tangent intercepted between the axes is of constant length. 6

OR

4. (a) Expand $\log x$ in powers of $(x-1)$ and hence evaluate $\log(1\cdot 1)$ correct up to 4 decimal places. 5

(b) For the curve $x = a[\cos t + \log \tan t/2]$, $y = a \sin t$, find the subtangent. 3

(c) If $v = (x^2 + y^2 + z^2)^{-1/2}$, then prove that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$$

6

5. (a) Find reduction formulae for $\int \sin^n x\,dx$ and $\int \cos^n x\,dx$. Hence show that

$$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx = \frac{(n-1)(n-3)(n-5)\cdots}{n(n-2)(n-4)\cdots} \times \left(\frac{\pi}{2}, \text{ only if } n \text{ is even}\right)$$

7

(b) Evaluate

$$\int_0^a \frac{x^n}{\sqrt{(a^2 - x^2)}}\,dx$$

Hence find the value of

$$\int_0^1 x^n \sin^{-1} x\,dx$$

7

OR

6. (a) Find reduction formulae for

$$\int \sin^m x \cos^n x\,dx$$

Hence show that

$$\int_0^{\pi/2} \sin^m x \cos^n x\,dx = \frac{(m-1)(m-3)\cdots(n-1)(n-3)\cdots}{(m+n)(m+n-2)(m+n-4)\cdots} \times \left(\frac{\pi}{2}, \text{ only if both } m \text{ and } n \text{ are even}\right)$$

7

(b) Find reduction formulae for

$$\int e^{ax} \sin^n x\,dx$$

Hence evaluate

$$\int e^x \sin^3 x\,dx$$

7

7. *(a)* Solve

$$\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$$

reducing it to homogeneous form. 7

(b) Solve : 3

$$\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx+\{x+\log x - x\sin y\}\,dy = 0$$

(c) Solve : 4

$$(x^2 - y^2)\,dx = xy\,dy$$

OR

8. *(a)* What is an exact differential equation? Prove that the necessary and sufficient condition for the differential equation to be exact is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ 7

(b) Solve : 3

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$$

(c) Solve : 4

$$\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$$

9. *(a)* Obtain differential equation of all circles of radius a and centre at (h, k). 5

(b) Solve $y'' + 4y' + 4y = 3\sin x + 4\cos x$, $y(0) = 1$ and $y'(0) = 0$. 6

(c) If y_1 and y_2 are two solutions of the equation

$$\frac{d^n y}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + a_2\frac{d^{n-2}y}{dx^{n-2}} + \cdots + a_n y = 0$$

then show that $c_1 y_1 + c_2 y_2 (\equiv u)$ is also a solution. 3

OR

10. *(a)* Solve : 5

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1 - e^x)^2$$

(b) Solve : 6

$$y'' - 4y' + 4y = 8x^2 e^{2x}\sin 2x$$

(c) Prove that

$$\frac{1}{D-a}X = e^{ax}\int Xe^{-ax}dx$$ 3

★ ★ ★