UG Odd Semester (CBCS) Exam., December—2016

B.Sc (Honours) B.Ed

MATHEMATICS

(Pass)

(3rd Semester)

Course No. : BSMP–303

(Classical Algebra, Trigonometry and Modern Algebra and Linear Algebra)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Let $A \quad M_n$ be Hermitian. Then prove that $x \;\; Ax$ is real for all $x \quad C^n$. 7

(b) Let B be an arbitrary nonsingular real square matrix. Then prove that

$$(\det B)^2 \quad {}^{\;n}_{i\;\;1}\;\;{}^{\;n}_{j\;\;1} |B_{ij}|^2$$ 7

OR

2. *(a)* Define conjugate transpose of a complex matrix. 5

(b) Find the conjugate transpose of the following complex matrix : 5

$$A \quad \begin{matrix} 3 \quad 7i & 0 \\ 2i & 4 \quad i \end{matrix}$$

(c) Define Hermitian matrix. 4

3. *(a)* If $y \quad e^{ax} \sin bx$, then prove that

$$y_2 \quad 2ay_1 \quad (a^2 \quad b^2)y \quad 0$$ 5

(b) Find the nth derivatives of

$$\frac{1}{1 \quad 5x \quad 6x^2}$$ 9

OR

4. *(a)* Prove that the sequence whose general term is $a_n \quad {}^{\;n}_{k\;\;0} \frac{1}{k!}$ converges. 7

(b) Find the $\lim\limits_{n} \; a_n$, where a_n is defined as $a_1 \quad 2$ and $a_{n \quad 1} \quad \frac{1}{2}(a_n \quad 6)$. 7

5. *(a)* What do you understand by De Moivre's theorem? 5

(b) Find all complex cube roots of $27i$. 9

OR

6. *(a)* Define hyperbolic sine function and prove that $\cosh x \quad \sinh x \quad e^{x}$. 6

(b) Integrate each of the following with respect to x : 2×4=8

(i) $\cos 3h$

(ii) $\sinh^{2} x$

(iii) $x \sinh x$

(iv) $e^{x} \cosh x$

7. *(a)* Explain the important properties of cyclic group. 7

(b) Let G be a group, $a \quad G$ such that $|a| \quad n$ and d a positive divisor of n. Then prove that $|a^{d}| \quad \frac{n}{d}$. 7

OR

8. *(a)* Let $f:(a, b) \quad R$. Suppose that $\underline{D} f(x)$ and $\overline{D} f(x)$ for every $x \quad (a, b)$. Then prove that the function f is continuous on the interval (a, b). 7

(b) Let $f:(a, b) \quad R$ be convex. Then prove that for $x_1, x_2 \quad (a, b)$ such that $x_1 \quad x_2$, it follows that $\overline{D} f(x_1) \quad \underline{D} f(x_2)$. 7

9. *(a)* Describe the properties of ring. 6

(b) Let D be a skew field with centre F. If D has an ordering p of higher level, then prove that F is algebraically closed in D. 8

OR

10. *(a)* Define vector space. 7

(b) Let V be vector space and $U \quad V$. If U is closed under vector addition and scalar multiplication, then show that U is a subspace of V. 7

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