PG Odd Semester (CBCS) Exam., December—2016

ECONOMICS

(1st Semester)

Course No. : EC–103 (C)

(Mathematical Methods for Economic Analysis)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Given

$$A = \begin{matrix} 7 & 1 \\ 6 & 9 \end{matrix}, \quad B = \begin{matrix} 0 & 4 \\ 3 & 2 \end{matrix} \text{ and } C = \begin{matrix} 8 & 3 \\ 6 & 1 \end{matrix}$$

Find *(i)* $3A$ and *(ii)* $4B+2C$.

(b) Determine the rank P of the following matrix :

$$B = \begin{matrix} 5 & 9 & 3 \\ 2 & 12 & 4 \\ 3 & 18 & 6 \end{matrix}$$

(c) Given the following model :

$Y \quad C \quad I_0 \quad G_0$
$C \quad a \quad b(Y \quad T) (a \quad 0,0 \quad b \quad 1)$
[t: taxes]
$T \quad d \quad tY (d \quad 0, 0 \quad t \quad 1)$
[t: income-tax rate]

(i) How many endogenous variables are there?

(ii) Find Y^*, T^* and C^*. 4+4+7=15

2. *(a)* What is the difference between a tangent line and a secant line? Illustrate with the help of a suitable diagram.

(b) Find all the first-order derivatives of the following functions :

(i) $y \quad (3x^2 \quad 13)^3$

(ii) $z \quad \left(\dfrac{8x \quad 7y}{5x \quad 2y}\right)^2$

(c) Find the values of the following integrals :

(i) $(2x^2 \quad 4x^6)\,dx$

(ii) $7e^{\,x} \quad \dfrac{2}{x}\,dx$

3+6+6=15

UNIT—II

3. *(a)* What is meant by inflection point of a function? Find the inflection point for the following function :

$$y \quad (x \quad 8)^4$$

(b) Find the relative maxima and minima of y by the second derivative test :

(i) $y \quad 2x^2 \quad 8x \quad 25$

(ii) $y \quad \frac{1}{3}x^3 \quad 3x^2 \quad 5x \quad 3$

(c) A firm has the following total cost and demand functions :

$$C \quad \frac{1}{3}Q^3 \quad 7Q^2 \quad 111Q \quad 50$$
$$Q \quad 100 \quad P$$

Find the maximum profit. 4+6+5=15

4. *(a)* Given the demand function $P_d \quad 113 \quad Q^2$ and supply function $P_s \quad (Q \quad 1)^2$. Find producer's surplus.

(b) Given

$$I_t \quad 4 \quad 2(Y_t \quad Y_{t \ 1})$$
$$S_t \quad 0 \quad 2Y_t$$
$$Y_o \quad 5600$$

Find the equilibrium level of income Y_t for any period. Also find the warranted rate of growth.

(c) Given demand and supply for the Cobweb model as follows. Find the intertemporal equilibrium price and determine whether the equilibrium is stable:

$$Q_{dt} \quad 18 \quad 3P_t$$
$$Q_{st} \quad 3 \quad 4P_{t \ 1}$$ 5+5+5=15

UNIT—III

5. *(a)* What is the significance of second-order test in optimization? Illustrate with an example.

(b) Find the extreme values, if any, of the following four functions. Check whether they are maxima or minima by the determinantal test :

(i) $Z \quad X_1^2 \quad 3X_2^2 \quad 3X_1X_2 \quad 4X_2X_3 \quad 6X_3^2$

(ii) $Z \quad X_1X_3 \quad X_1^2 \quad X_2 \quad X_2X_3 \quad X_2^2 \quad 3X_3^2$

(c) A utility function is given by the equation

$$U \quad x \quad 2x^2 \quad xy \quad 40y \quad y^2$$

where x is the number of units of good x and y is the number of units of good y consumed. Calculate the maximum level of utility. 4+6+5=15

6. *(a)* A firm produces two products which are sold in two separate markets with the demand schedules.

$$P_1 \quad 600 \quad 0 \;3q_1$$
$$P_2 \quad 500 \quad 0 \;2q_2$$

Production costs are related and the firm faces the total cost function

$$TC \quad 16 \quad 1 \;2q_1 \quad 1 \;5q_2 \quad 0 \;2q_1 q_2$$

If the firm wishes to maximize total profits, how much of each product should it sell? What will be the maximum level of profit?

(b) A multiplant monopoly operates two plants whose total cost schedules are

$$TC_1 \quad 8 \;5 \quad 0 \;03q_1^2$$
$$TC_2 \quad 5 \;2 \quad 0 \;04q_2^2$$

If it faces the demand schedule

$$P \quad 60 \quad 0 \;04q$$

where $q \quad q_1 \quad q_2$, how much should it produce in each plant in order to maximize profits? 8+7=15

UNIT—IV

7. *(a)* Write down the economic interpretation of Lagrange multiplier.

(b) If, in stead of $g(x, y) \quad C$, the constraint is written in the form of $G(x, y) \quad 0$, then how should the Lagrangian function and the first-order condition be modified as a consequence?

(c) *(i)* What output mix should a profit-maximizing firm produce when its total profit function is $80x \quad 2x^2 \quad xy \quad 3y^2 \quad 100y$ and its maximum output capacity is $x \quad y \quad 12$?

(ii) Estimate the effect on profits if output capacity is expanded by 1 unit. 5+4+6=15

8. *(a)* A small publishing company decides to use one section of its plant to produce two textbooks called *A* and *B*. The profit made on each copy is ₹12 for *A* and ₹ 18 for *B* . Each copy of book *A* requires 12 minutes for printing and 18 minutes for binding. The corresponding figures for book *B* are 15 and 9 minutes respectively. There are 10 hours available for printing and 10·5 hours are available for binding. How many of each should be produced to maximize profit?

(b) Solve the linear programming problem by simplex method :

Maximize $Z \quad 4x \quad 9y$

subject to

$$\begin{aligned} 5x \quad 3y \quad 30 \\ 7x \quad 2y \quad 28 \\ x \quad 0, y \quad 0 \end{aligned}$$

8+7=15

UNIT—V

9. *(a)* Distinguish between zero-sum game and non-zero-sum game.

(b) Determine the optimal strategies for the two players *A* and *B* and find the value of the game from the following payoff matrix :

		Player A			
Player B		3	–1	4	2
		–1	–3	–7	0
		4	–6	2	–9

(c) Given the payoff matrix

		Firm–B	
		Low Price	*High Price*
Firm–A	*Low Price*	1, 1	3, –1
	High Price	–1, 3	4, 2

where the payoffs are the profits or losses of the tow forms.

(i) Do both the firms have a dominant strategy?

(ii) What is the optimal strategy for each firm? Also find the Nash equilibrium, if there is one.

4+5+6=15

10. *(a)* What is meant by sequential game? Given the following payoff matrix :

		Strategies of firm 2	
		S_1	S_2
Strategies of Firm 1	S_1	{ 2, 2 }	{ 3, 6 }
	S_2	{ 6, 3 }	{ 2, 2 }

(i) Represent the above game through its extensive form.

(ii) Do you think it is the first mover that gains more from the game? Justify your answer.

(b) Consider the tit-for-tat strategy in the repeated prisoner's dilemma. Suppose that one player makes a mistake and defects when he meant to cooperate. If both players continue to play tit-for-tat after that, what happens?

(c) Explain how the law that banned cigarette advertising on televison in India solved the prisonen's dilemma for cigarette producers. (2+2+2)+5+4=15

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