PG Odd Semester (CBCS) Exam., December—2016

ECONOMICS

(1st Semester)

Course No. : ECOCC–103

(Mathematical Methods for Economic Analysis)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Let $A \quad \{1, 2, 3, 5\}$ and $B \quad \{4, 6, 9, 10\}$. Define a relation R from A to B by

$$R \quad \{(x, y) : x \quad y \quad \text{odd number};\ x \quad A,\ y \quad B\}$$

(b) Given

$$A \quad \begin{matrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{matrix}\ ,\ B \quad \begin{matrix} 2 & 0 \\ 3 & 8 \end{matrix} \ \text{and}\ C \quad \begin{matrix} 7 & 2 \\ 6 & 3 \end{matrix}$$

(i) Is AB defined? Calculate AB. Can you calculate BA? Why?

(ii) Is BC defined? Calculate BC. Is CB defined? If so, calculate CB. Is it true that $BC \quad CB$?

(c) Given the IS equation

$$0\ 3Y \quad 100i \quad 252 \quad 0$$

and the LM equation

$$0\ 25Y \quad 200i \quad 176 \quad 0$$

Find the equilibrium level of income (Y) and rate of interest (i). 2+(3+4)+5=14

2. *(a)* Find all the first-order derivatives of the following functions :

(i) $Y \quad (3x^2 \quad 13)^3$

(ii) $Z \quad \dfrac{8x \quad 7y}{5x \quad 2y}^{\,2}$

(b) Find the following :

(i) $(x \quad 3)(x \quad 1)^{1/2}\, dx$

(ii) $x \ln x\, dx\ (x \quad 0)$

(c) Solve the following equation :

$$\frac{dy}{dt} \quad 6y \quad 18$$

Is the system dynamically stable? Comment. 5+5+4=14

UNIT—II

3. *(a)* Which of the following quadratic functions is strictly convex?

(i) $Y \quad 9x^2 \quad 4x \quad 8$

(ii) $V \quad 9 \quad 2x^2$

(iii) $W \quad\quad 3x^2 \quad 39$

(b) Find the value of x at which the following function attains inflection point :

$$y \quad x^3 \quad 18x^2 \quad 96x \quad 80$$

(c) Find the marginal cost (MC) function for the following average cost (AC) function :

$$\text{AC} \quad 1 \ 5Q \quad 4 \quad \frac{46}{Q}$$

6+4+4=14

4. *(a)* Given the following demand (P_d) and supply (P_s) functions :

$$P_d \quad 113 \quad Q^2$$
$$P_s \quad (Q \quad 1)^2$$

Find the producer's surplus.

(b) Given

$$I_t \quad 4 \ \ 2(Y_t \quad Y_{t \ \ 1})$$
$$S_t \quad 0 \ \ 2Y_t$$
$$Y_0 \quad 5600$$

Find the equilibrium level of income Y_t for any period. Also calculate the rate of warranted growth.

(c) Given demand and supply for the Cobweb model

$$Q_{dt} \quad 18 \quad 3P_t$$
$$Q_{st} \quad\quad 3 \quad 4P_{t \ \ 1}$$

(i) Find the intertemporal equilibrium price.

(ii) Determine whether the equilibrium is stable.

5+4+5=14

UNIT—III

5. *(a)* What is the significance of second-order test in optimization? Illustrate.

(b) Find the optimum value of y for the following functions :

(i) $y \quad\quad 2x^2 \quad 8x \quad 25$

(ii) $y \quad \frac{1}{3}x^3 \quad 3x^2 \quad 5x \quad 3$

(c) Given the utility function

$$U \quad x \quad 2x^2 \quad xy \quad 40y \quad y^2$$

where x is the number of good X and y is the number of good Y consumed. Determine the value of x and y for which U is maximum. 5+5+4=14

6. *(a)* A firm produces two products which are sold in two separate markets with the demand schedules

$$P_1 \quad 600 \quad 0 \; 3q_1 \text{ and } P_2 \quad 500 \quad 0 \; 2q_2$$

The firm faces the total cost (TC) function

$$\text{TC} \quad 16 \quad 1 \; 2q_1 \quad 1 \; 5q_2 \quad 0 \; 2q_1q_2$$

If the firm wishes to maximize total profits, how much of each product should itself? What will the maximum profit level be? Discuss.

(b) A multiplant monopoly operates two plants whose total cost schedules are

$$TC_1 \quad 8 \; 5 \quad 0 \; 03q_1^2$$

$$TC_2 \quad 5 \; 2 \quad 0 \; 04q_2^2$$

If it faces the demand schedule

$$P \quad 60 \quad 0 \; 04q$$

where $q_1 \quad q_2 \quad q$, how much should it produce in each plant to maximize profits? 7+7=14

UNIT—IV

7. *(a)* What is constraint optimization? Illustrate with a suitable example.

(b) If, instead of $g(x, y) \quad c$, the constraint is written in the form of $G(x, y) \quad 0$, how should the Lagrangian function and the first-order condition be modified as a consequence?

(c) A firm has the production function $Q \quad K^{0 \; 5}L^{0 \; 5}$ and buys input K at ₹ 12 a unit and input L at ₹ 3 a unit and has a budget of ₹ 600. Use the Lagrangian method to find the input combination that maximizes output. Also check the second-order condition for optimization. 5+3+6=14

8. *(a)* A company manufactures two types of wrought gates. The number of man hours required to produce each type of gates along with the maximum number of hours available is given in the following table :

	Welding	*Finishing*	*Admin.*	*Selling Price* (₹)
Type I gate	6	2	1	120
Type II gate	2	1	1	95
Maximum hours available	840	300	250	

Table 1 : Requirement for Gates Type I and II

(i) Express the information given on the requirements in terms of inequality constraints.

(ii) Graph the inequality constraints and shade in the feasible region.

(iii) Calculate the corner points of the feasible region.

(b) A primal problem is given as

$$\text{Minimize} \quad C \quad x_1 \quad 4x_2$$

subject to

$$\begin{matrix} 1 & 2 \\ 3 & 2 \end{matrix} \quad \begin{matrix} x_1 \\ x_2 \end{matrix} \quad \begin{matrix} 8 \\ 12 \end{matrix}$$

$$x_1, x_2 \quad 0$$

Write its dual programme and solve the problem. 7+7=14

UNIT—V

9. *(a)* Explain the meaning of a Nash equilibrium. How does it differ from an equilibrium in dominant strategies?

(b) Suppose your opponent is not playing his/her Nash equilibrium strategy. Should you play your Nash equilibrium strategy? Explain.

(c) Given the following payoff matrix :

		Firm B	
		Low Price	*High Price*
Firm A	*Low Price*	1, 1	3, 1
	High Price	1, 3	4, 2

Table 2 : Payoff Matrix

(i) Do both the firms have a dominant strategy?

(ii) Find the optimal strategy for each firm, and the Nash equilibrium, if there is one. 4+5+5=14

10. *(a)* Illustrate the basic idea of coordination game with a special reference to battle of sexes.

(b) Given the following payoff matrix :

		Younger Generation	
		Support	*Refrain*
Older Generation	*Save*	3, 1	1, 0
	Squander	2, 1	2, 2

Table 3 : Intergenerational Conflict over Saving

Determine the optimal strategy for each of the two players assuming that older generation move first. Do you have any policy implication for this game? Briefly state. 7+(5+2)=14

★ ★ ★